Dynamics and Self-Similarity in Min-Driven Clustering

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Abstract: We study a mean-field model for a clustering process that may be described informally as follows. At each step a random integer k is chosen with probability p_k, and the smallest cluster merges with k randomly chosen clusters. We prove that the model determines a continuous dynamical system on the space of probability measures supported in $(0, \infty)$, and we establish necessary and sufficient conditions for approach to self-similar form. We also characterize eternal solutions for this model via a Lévy-Khintchine formula. The analysis is based on an explicit solution formula discovered by Gallay and Mielke, extended using a careful choice of time scale.