Solutions of Neumann problems in domains with cracks and applications to fracture mechanics

The first part of the course deals with the study of solutions of the Laplace equation in $\Omega \setminus K$, where Ω is a two dimensional smooth domain and K is a connected compact subset of Ω. The solutions are required to satisfy a homogeneous Neumann boundary condition on K and a nonhomogeneous Dirichlet condition on (part of $\partial\Omega$. The main result is the continuous dependence of the solution on K, with respect to the Hausdorff metric. Classical examples show that the result is no longer true without the hypothesis that K is connected.

The second part of the course uses this stability result to give a rigorous mathematical formulation of a variational quasistatic model for the slow growth of a brittle fracture, recently introduced by Francfort and Marigo. Starting from a discrete time formulation, a more satisfactory continuous time formulation is obtained, with full justification of the limit argument. Griffith's criterion for fracture growth is stated and proved within this model.