# Math 292: Homework 7

Due Wednesday March 30 in lecture

Let $A$, $\mathbf{b}$ and $\mathbf{c}$, as well as $\alpha_1, \cdots, \alpha_n$, $\beta_1, \cdots, \beta_m$ be fixed. We say that a function $h : \mathbb{R} \to \mathbb{R}$ is *convex* if for all $x, y$ and $\alpha \in [0, 1]$ we have

$$\alpha h(x) + (1-\alpha)h(y) \geq h\left(\alpha x + (1-\alpha)y\right).$$

We say that a function $h$ is concave if $-h$ is convex.

1. Let $Z^*(\theta)$ be the optimal value of the objective function for the following linear optimization problem:

$$\begin{aligned} \max \quad & (c_1 + \alpha_1\theta)x_1 + \cdots + (c_n + \alpha_n\theta)x_n \\ \text{subject to} \quad & A\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

   Show that $Z^*(\theta)$ is convex.

2. Let $\hat{Z}(\theta)$ be the optimal value of the objective function for the following:

$$\begin{aligned} \max \quad & c_1x_1 + \cdots + c_nx_n \\ \text{subject to} \quad & \sum_{j=1}^{n} a_{ij}x_j \leq b_i + \beta_i\theta \qquad 1 \leq i \leq m \\ & \mathbf{x} \geq \mathbf{0}. \end{aligned}$$

   Show that $\hat{Z}(\theta)$ is concave.