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Problem #7.1-1

(a)

Optimal solution is

$(x_1, x_2, x_3) = (\frac{5}{3}, 0, 3)$ $Z = 17$

(b)

Dual Problem

minimize $Z^* = 25y_1 + 20y_2$

st $6y_1 + 3y_2 \geq 3$

$3y_1 + 4y_2 \geq 1$

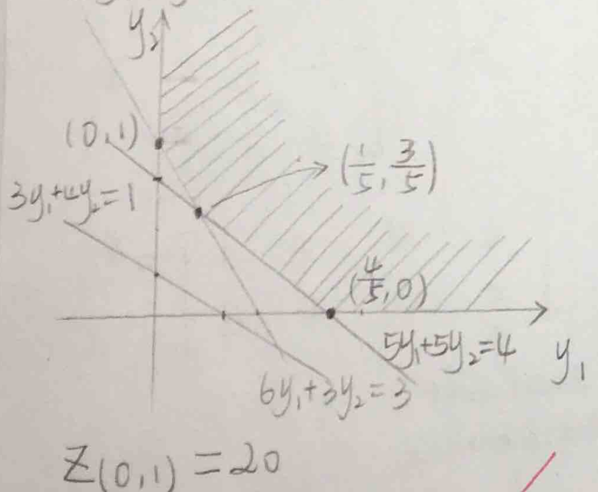
$5y_1 + 5y_2 \geq 4$

$y_1, y_2 \geq 0$

(c)

Optimal Solution

$(y_1, y_2) = (\frac{1}{5}, \frac{3}{5})$



$Z(0,1) = 20$

$Z(\frac{1}{5}, \frac{3}{5}) = 17$ ✓

$Z(\frac{4}{5}, 0) = 20$

(d) $6 \times \frac{1}{5} + 3 \times \frac{3}{5} \geq 3$

$2 \times \frac{1}{5} + 3 \times \frac{3}{5} = \frac{11}{5} < 3$ X

$5 \times \frac{1}{5} + 5 \times \frac{3}{5} \geq 4$ It is not optimal anymore.

(e)

$$S^* = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix}$$

$$-C^* = [-3 \ -3 \ -4]$$

$$y^* = [\tfrac{1}{5} \ \tfrac{3}{5}]$$

$$y^*(A+\Delta A) - C^* = [\tfrac{1}{5} \ \tfrac{3}{5}] \begin{bmatrix} 6 & 2 & 5 \\ 3 & 3 & 5 \end{bmatrix} + [-3 \ -3 \ -4]$$

$$= [3 \ \tfrac{11}{5} \ 4] + [-3 \ -3 \ -4]$$

$$= [0 \ -\tfrac{4}{5} \ 0]$$

$$S^*(A+\Delta A) = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix} \begin{bmatrix} 6 & 2 & 5 \\ 3 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & \frac{4}{5} & 1 \end{bmatrix}$$

$$y^* b^* = [\tfrac{1}{5} \ \tfrac{3}{5}] \begin{bmatrix} 25 \\ 20 \end{bmatrix} = 17$$

$$S^* b^* = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix} \begin{bmatrix} 25 \\ 20 \end{bmatrix} = \begin{bmatrix} \frac{5}{3} \\ 3 \end{bmatrix}$$

$$\begin{matrix} 1 & 0 & -\frac{4}{5} & 0 & \frac{1}{5} & \frac{3}{5} & 17 \\ 0 & 1 & -\frac{1}{3} & 0 & \frac{1}{3} & -\frac{1}{3} & \frac{5}{3} \\ 0 & 0 & \frac{4}{5} & 1 & -\frac{1}{5} & \frac{2}{5} & 3 \end{matrix}$$

After adjusting, coefficients are $-\frac{4}{5}$ $-\frac{1}{3}$ $\frac{4}{5}$

(f)

Dual Problem

minimize $Z^* = 25y_1 + 20y_1$

st $6y_1 + 3y_2 \geq 3$

$3y_1 + 4y_2 \geq 1$

$5y_1 + 5y_2 \geq 4$

$3y_1 + 2y_2 \geq 2$

$y_1, y_2 \geq 0$

$6 \times \frac{1}{5} + 3 \times \frac{3}{5} \geq 3$

$3 \times \frac{1}{5} + 4 \times \frac{3}{5} \geq 1$

$5 \times \frac{1}{5} + 5 \times \frac{3}{5} \geq 4$

$3 \times \frac{1}{5} + 2 \times \frac{3}{5} < 2$ X

Thus the original solution is not optimal any more ✓

(g)

$S^* = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix}$ $y^* = \begin{bmatrix} \frac{1}{5} & \frac{3}{5} \end{bmatrix}$

$A^* = \begin{bmatrix} 6 & 3 & 5 & 3 \\ 3 & 4 & 5 & 2 \end{bmatrix}$

this last column matters

$C^* = \begin{bmatrix} 3 & 1 & 4 & 2 \end{bmatrix}$

this last column matters

Last column in S^*A^* is $\begin{bmatrix} \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$ Last column in $y^*A^* - C^*$ is $-\frac{1}{5}$

thus coefficients are $-\frac{1}{5}$ $\frac{1}{3}$ $\frac{1}{5}$.

Problem #7.2-2

$S^* = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}$ $y^* = [5 \ 0]$

$A = \begin{bmatrix} -1 & 1 & 3 \\ 12 & 4 & 10 \end{bmatrix}$ $C = [-5 \ 5 \ 13]$ $b = \begin{bmatrix} 20 \\ 90 \end{bmatrix}$

(a) $\Delta b = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$ $y^*\Delta b = 50$ $S^*\Delta b = \begin{bmatrix} 10 \\ -40 \end{bmatrix}$

	Z	X_1	X_2	X_3	X_4	X_5	RHS
Z	1	0	0	2	5	0	150
X_2	0	-1	1	3	1	0	30
X_5	0	16	0	-2	-4	1	-30

It is super-optimal: $X_5 = -30 < 0$ (violates nonnegativity)

(b) $\Delta b = \begin{bmatrix} 0 \\ -20 \end{bmatrix}$ $y^*\Delta b = 0$ $S^*\Delta b = \begin{bmatrix} 0 \\ -20 \end{bmatrix}$

	Z	X_1	X_2	X_3	X_4	X_5	RHS
Z	1	0	0	2	5	0	100
X_2	0	-1	1	3	1	0	20
X_5	0	16	0	-2	-4	1	-10

It is super-optimal $X_5 = -10 < 0$ (violates non-negativity)

(c) $\Delta b = \begin{bmatrix} -10 \\ 10 \end{bmatrix}$ $y^*\Delta b = -50$ $S^*\Delta b = \begin{bmatrix} -10 \\ 50 \end{bmatrix}$

	Z	X_1	X_2	X_3	X_4	X_5	RHS
Z	1	0	0	2	5	0	50
X_2	0	-1	1	3	1	0	10
X_5	0	16	0	-2	-4	1	60

The solution is feasible and optimal

(d)

$\Delta C = [0 \quad 0 \quad -5]$.

thus we have

	Z	X_1	X_2	X_3	X_4	X_5	RHS
Z	1	0	0	7	5	0	100
X_2	0	-1	1	3	1	0	20
X_5	0	16	0	-2	-4	1	10

The solution is both feasible and optimal

(e) $\Delta C = [3 \quad 0 \quad 0]$ $\quad \Delta A = \begin{bmatrix} 1 & 0 & 0 \\ -7 & 0 & 0 \end{bmatrix}$ $\quad y^* \Delta A = [5 \quad 0] \begin{bmatrix} 1 & 0 & 0 \\ -7 & 0 & 0 \end{bmatrix} = [5 \quad 0 \quad 0]$

$C_{new} = [2 \quad 0 \quad 2]$ $\quad S^* \Delta A = \begin{bmatrix} 1 & 0 & 0 \\ -11 & 0 & 0 \end{bmatrix}$

	Z	X_1	X_2	X_3	X_4	X_5	RHS
Z	1	2	0	2	5	0	100
X_2	0	0	1	3	1	0	20
X_5	0	5	0	-2	-4	1	10

The solution is both feasible and optimal

(f) $\Delta C = [0 \quad 1 \quad 0]$ $\quad \Delta A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ $\quad y^* \Delta A = [0 \quad 5 \quad 0]$

$C_{new} = [0 \quad 4 \quad 2]$ $\quad S^* \Delta A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -3 & 0 \end{bmatrix}$

	Z	X_1	X_2	X_3	X_4	X_5	RHS
Z	1	0	4	2	5	0	100
X_2	0	-1	2	3	1	0	20
X_5	0	16	-3	-2	-4	1	10

↓ Row operation

	Z	X_1	X_2	X_3	X_4	X_5	RHS
Z	1	0	4	2	5	0	100
X_2	0	-0.5	1	1.5	0.5	0	10
X_5	0	-14.5	0	2.5	-2.5	1	40

The solution is both feasible and optimal.

1(g) $\Delta c = [0\ 0\ 0\ 10]$ $y^* \Delta A = [0\ 0\ 0\ 15]$ $C_{new} = [0\ 0\ 2\ 5]$

$\Delta A = \begin{bmatrix} 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix}$ $S^* \Delta A = \begin{bmatrix} 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & -7 \end{bmatrix}$

	Z	X_1	X_2	X_3	X_6	X_4	X_5	RHS
Z	1	0	0	2	5	5	0	100
X_2	0	-1	1	3	3	1	0	20
X_5	0	16	0	-2	-7	-4	1	10

The solution is both feasible and optimal. ✓

(h)

	Z	X_1	X_2	X_3	X_4	X_5	X_6	RHS
Z	1	0	0	2	5	0	0	100
X_2	0	-1	1	3	1	0	0	20
X_5	0	16	0	-2	-4	1	0	10
X_6	0	2	3	5	0	0	1	50

↳ Row operation

	Z	X_1	X_2	X_3	X_4	X_5	X_6	RHS
Z	1	0	0	2	5	0	0	100
X_2	0	-1	1	3	1	0	0	20
X_5	0	16	0	-2	-4	1	0	10
X_6	0	5	0	-4	-3	0	1	-10

The solution is super-optimal since $X_6 = -10$ (violates non-negativity) ✓

Not feasible not optimal.

(i)

$$\Delta A = \begin{bmatrix} 0 & 0 & 0 \\ -2 & -1 & 0 \end{bmatrix} \quad \Delta b = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

$$y^* \Delta A = [0 \ 0 \ 0] \quad y^* \Delta b = 0$$

$$S^* \Delta A = \begin{bmatrix} 0 & 0 & 0 \\ -2 & -1 & 0 \end{bmatrix} \quad S^* \Delta b = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

	Z	X_1	X_2	X_3	X_4	X_5	RHS
Z	1	0	0	2	5	0	100
X_2	0	-1	1	3	1	0	20
X_5	0	14	1	-2	-4	1	20

Row operation

	Z	X_1	X_2	X_3	X_4	X_5	RHS
Z	1	0	0	2	5	0	100
X_2	0	-1	1	3	1	0	20
X_5	0	15	0	-5	-5	1	0

The solution is feasible and optimal

Graded for completion.

Problem #7.2-6

(a) $\Delta b = \begin{bmatrix} -5 \\ 1 \\ -2 \end{bmatrix}$ $S^* = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$ $y^* = [1 \ 1 \ 0]$

$S^* \Delta b = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$ $y^* \Delta b = -4$

	Z	X_1	X_2	X_3	X_4	X_5	X_6	RHS
Z	1	0	0	2	1	1	0	14
X_2	0	0	1	5	1	3	0	22
X_6	0	0	0	2	0	1	1	8
X_1	0	1	0	4	1	2	0	18

It is both feasible and optimal

(b) $\Delta C = [0 \ 0 \ 1]$

	Z	X_1	X_2	X_3	X_4	X_5	X_6	RHS
Z	1	0	0	1	1	1	0	18
X_2	0	0	1	5	1	3	0	24
X_6	0	0	0	2	0	1	1	7
X_1	0	1	0	4	1	2	0	21

It is both feasible and optimal

(c) $\Delta C = [1 \ 0 \ 0]$

	Z	X_1	X_2	X_3	X_4	X_5	X_6	RHS
Z	1	-1	0	2	1	1	0	18
X_2	0	0	1	5	1	3	0	24
X_6	0	0	0	2	0	1	1	7
X_1	0	1	0	4	1	2	0	21

Row operation

	Z	X_1	X_2	X_3	X_4	X_5	X_6	RHS
Z	1	0	0	6	2	3	0	39
X_2	0	0	1	5	1	3	0	24
X_6	0	0	0	2	0	1	1	7
X_1	0	1	0	4	1	2	0	21

It is both feasible and optimal

(d) $\Delta C = [0 \ 0 \ 3]$

$\Delta A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 8 & 0 & 0 \end{bmatrix}$ $S^* \Delta A = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ $y^* \Delta A = [0 \ 0 \ 2]$

	Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
Z	1	0	0	1	1	1	0	18
x_2	0	0	1	9	1	3	0	24
x_6	0	0	0	3	0	1	1	7
x_1	0	1	0	7	1	2	0	21

It is both feasible and optimal.

(e) $\Delta C = [-1 \ \ -1 \ \ 0]$

$$\Delta A = \begin{bmatrix} -2 & 0 & 0 \\ -1 & 2 & 0 \\ 2 & 3 & 0 \end{bmatrix}$$

$$y^* \Delta A = [-3 \ \ 2 \ \ 0]$$

$$S^* \Delta A = \begin{bmatrix} -5 & 6 & 0 \\ 1 & 5 & 0 \\ -4 & 4 & 0 \end{bmatrix}$$

	Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
Z	1	-2	3	2	1	1	0	18
x_2	0	-5	7	5	1	3	0	24
x_6	0	1	5	2	0	1	1	7
x_1	0	-3	4	4	1	2	0	21

↓

	Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
Z	1	0	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	0	4
x_2	0	0	$\frac{1}{3}$	$-\frac{5}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	-11
x_6	0	0	$\frac{19}{3}$	$\frac{10}{3}$	$\frac{1}{3}$	$\frac{5}{3}$	1	14
x_1	0	1	$-\frac{4}{3}$	$-\frac{4}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	-7

→

	Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
Z	1	0	0	1	1	0	0	15
x	0	0	1	-5	-2	-1	0	-33
x	0	0	0	-25	13	8	1	223
x	0	1	0	-8	-3	-2	0	-51

It is super-optimal

(f) $\Delta C = [3 \ 2 \ 2]$

	Z	X_1	X_2	X_3	X_4	X_5	X_6	RHS
Z	1	-3	-2	0	1	1	0	18
X_2	0	0	1	5	1	3	0	24
X_6	0	0	0	2	0	1	1	7
X_1	0	1	0	4	1	2	0	21

	Z	X_1	X_2	X_3	X_4	X_5	X_6	RHS
Z	1	0	0	22	6	13	0	129
X_2	0	0	1	5	1	3	0	24
X_6	0	0	0	2	0	1	1	7
X_1	0	1	0	4	1	2	0	21

It is both feasible and optimal.

(g) $\Delta b = \begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix}$ $\quad S^*\Delta b = \begin{bmatrix} -3 \\ 0 \\ -3 \end{bmatrix}$ $\quad y^*\Delta b = -3$

$\Delta A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad S^*\Delta A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & 0 & 0 \\ -1 & 1 & 2 \end{bmatrix}$ $\quad y^*\Delta A = [-1 \ 1 \ 2]$

	Z	X_1	X_2	X_3	X_4	X_5	X_6	RHS
Z	1	-1	1	4	1	1	0	15
X_1	0	-1	2	7	1	3	0	21
X_6	0	0	0	2	0	1	1	7
X_2	0	0	1	6	1	2	0	18

$\rightarrow$

	Z	X_1	X_2	X_3	X_4	X_5	X_6	RHS
Z	1	0	0	3	1	0	0	12
X_1	0	1	0	5	1	1	0	15
X_6	0	0	0	2	0	1	1	7
X_2	0	0	1	6	1	2	0	18

It is both optimal and feasible