21-292 Midterm I – Spring 2016
Prof. Shlomo Ta'asan

Name:______________________
Section:______________________

Problem 1 (30 points):

Slim-Down Manufacturing makes a line of nutritionally complete, weight-reduction beverages. One of their products is a strawberry shake which is designed to be a complete meal. The strawberry shake consists of several ingredients. Some information about each of these ingredients is given below.

Ingredient	Calories from fat (per tbsp)	Total Calories (per tbsp)	Vitamin Content (mg/tbsp)	Thickeners (mg/tbsp)	Cost (¢/tbsp)
Strawberry flavoring	1	50	20	3	10
Cream	75	100	0	8	8
Vitamin supplement	0	0	50	1	25
Artificial sweetener	0	120	0	2	15
Thickening agent	30	80	2	25	6

The nutritional requirements are as follows. The beverage must total between 380 and 420 calories (inclusive). No more than 20% of the total calories should come from fat. There must be at least 50 milligrams (mg) of vitamin content. For taste reasons, there must be at least two tablespoons (tbsp) of strawberry flavoring for each tbsp of artificial sweetener. Finally, to maintain proper thickness, there must be exactly 15 mg of thickeners in the beverage.

Management would like to select the quantity of each ingredient for the beverage which would minimize cost while meeting the above requirements.

Formulate a linear programming model for this problem.

Problem 2 (30 points):

Consider the following problem:

Maximize $Z = 2x_1 + 4x_2 - x_3$,

subject to

$$\begin{aligned} 3x_2 - x_3 &\le 30 && \text{(resource 1)}\\ 2x_1 - x_2 + x_3 &\le 10 && \text{(resource 2)}\\ 4x_1 + 2x_2 - 2x_3 &\le 40 && \text{(resource 3)} \end{aligned}$$

and

$$x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0.$$

Work through the simplex method step by step to solve the problem.

Problem 3 (30 pts):

Consider the following problem.

$$\textbf{Maximize} \qquad Z = 2x_1 + 4x_2 + 3x_3,$$

subject to

$$x_1 + 3x_2 + 2x_3 = 20$$
$$x_1 + 5x_2 \geq 10$$

and

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$$

Perform Phase I of the two phase method. Then set up the table for Phase II. Is the solution optimal? <u>Do not perform additional Phase II simplex iterations!</u>

Problem 4 (10 pts)

Let the function $\mathbf{f : R \rightarrow R}$ satisfy

$$f(t x + (1-t) y) \leq t f(x) + (1-t) f(y) \qquad \text{for all } x,y \ \varepsilon \ R \text{ and } 0 \leq t \leq 1$$

Show that the set $\mathbf{C = \{x \mid f(x) \leq a \}}$ is a convex set.