21-292 OPERATIONS RESEARCH I -SPRING 2017
FINAL EXAM
PROF. SHLOMO TA'ASAN

Name:

Section:

Note:
1. There are 9 problems in total. You need to
   a. do Problem 1, 2, 3, 4
   b. do 3 out of 5 from Problem 5-9 and circle which problems you did in the table below
2. Show your steps
3. Use table method in simplex method.

	Points
P1	
P2	
P3	
P4	
P5	
P6	
P7	
P8	
P9	
Total	

Problem 1 (16 pts)**.** Consider the following linear programming problem:

$$\begin{aligned} \text{maximize: } & Z = x_1 + 2x_2 + 3x_3 + 4x_4 \\ \text{subject to: } & x_1 + 2x_2 + 2x_3 + 3x_4 \leq 24 \\ & 2x_1 + x_2 + 3x_3 + 2x_4 \leq 16 \\ & x_1,\ x_2,\ x_3,\ x_4 \geq 0 \end{aligned}$$

1. Find an optimal solution by using simplex method.
2. Determine the dual problem and its solution without using simplex method. (justify your answer)

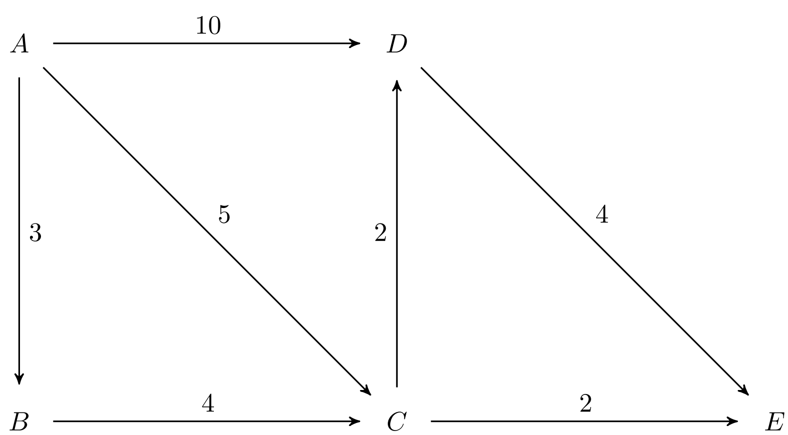
Problem 2 (16 pts)**.** Consider the following convex programming problem, where $f(\mathbf{x})$ is known to be concave.

$$\begin{aligned} \text{maximize: } & f(\mathbf{x}) = 2x_1 - 2x_1^2 + 2x_1x_2 - \frac{1}{2}x_2^2 + 8x_2 \\ \text{subject to: } & x_1 + 2x_2 \leq 8 \end{aligned}$$

where $x_1 \geq 0$ and $x_2 \geq 0$.

1. State the **KKT** conditions for above quadratic programming problem, and demonstrate that $(x_1, x_2) = (1, 1)$ is not optimal.
2. Use the modified simplex method to derive an optimal solution. (please indicate clearly the enter/leaving variable at each step)

Problem 3 (16 pts)**.** Consider the minimum cost flow problem given below. The c_{ij} values (cost per unit flow) are given on each arc



Arc capacities:

$A \longrightarrow C:\ 13$ $\qquad$ $C \longrightarrow E:\ 80$ $\qquad$ Other: $+\infty$

Supply and Demand:

$$b_A = 40,\ b_B = 50,\ b_C = 0,\ b_D = -30,\ b_E = -60$$

1. You are given that AD, DE, BC, and CE are basic arcs. Obtain an initial BF solution by solving the feasible spanning tree with those basic arcs.
2. Determine the next entering basic variable (arc), the leaving basic variable (arc), and the BF solution. Please show your steps.
3. Based on the BF solution obtained from 2, determine the next entering basic variable (arc), the leaving basic variable (arc), and the BF solution. Also, indicate how the network has to be adjusted according to this solution. Please show your steps.
4. Continue until you finish.

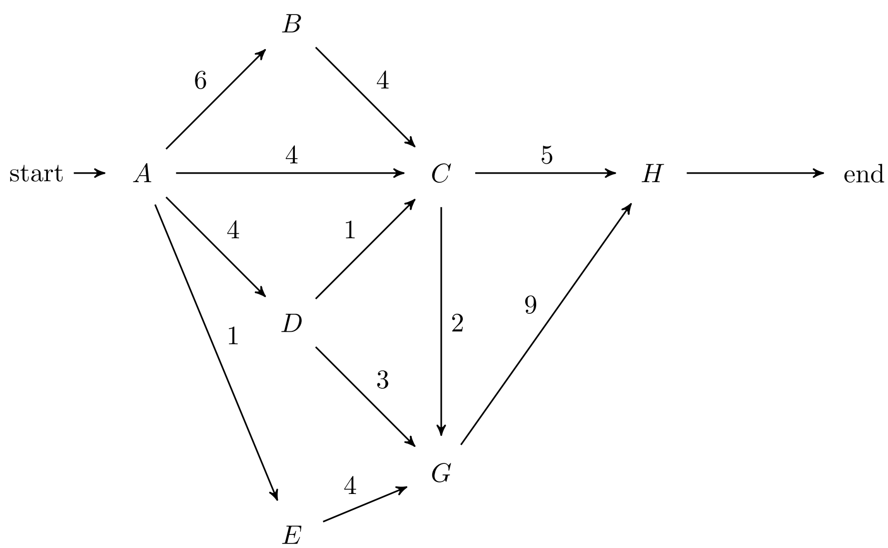
Problem 4 (16 pts)**.**

		Destination				
		1	**2**	**3**	**4**	**Supply**
	1	3	7	6	4	5
Source	2	2	4	3	2	2
	3	4	3	M	5	3
Demand		3	3	2	2	

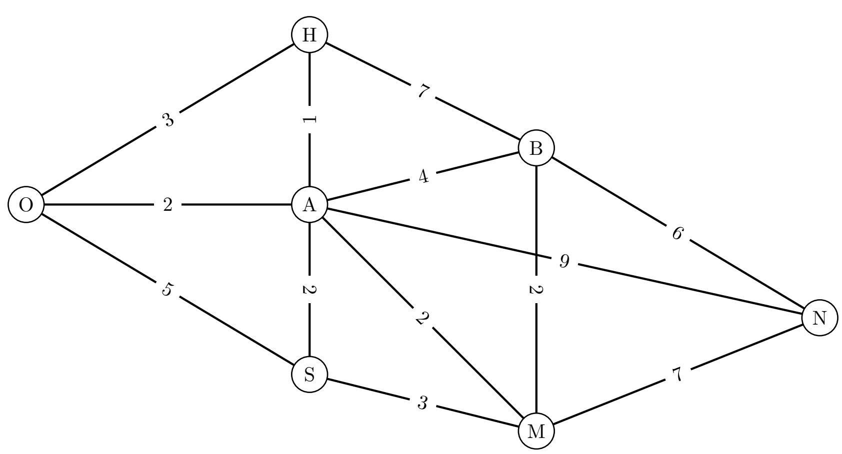
where M denotes a large integer.

1. Use Northwest corner rule to find an initial solution. Continue the transportation simplex method until you finish.
2. Use Vogel's approximation method to find an initial solution. Determine whether the solution is optimal and, if so, why.
3. State the advantage/disadvantage for Northwest corner rule and Vogel's approximation

Problem 5 (12 pts)**.** Find the flow pattern giving the *maximum flow* from Node A to Node H in the following network problem. Show all steps involved.



Problem 6 (12 pts)**.** Consider the following network.



Find all possible *shortest paths* from Node O to Node N. Construct the paths using a tabular form as follows.

Step	Solved Nodes Directly Connected to Unsolved Nodes	Closest Connected Unsolved Node	Total Distance Involved	nth Nearest Node	Minimum Distance	Last Connection

Problem 7 (12 pts)**.** Consider the following linear programming problem

$$\text{minimize: } Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$
$$\text{subject to: } 0 < a \leq x_i \leq b,$$

for $i = 1, \ldots, n$ and $a < b$.

1. Derive an optimal solution and prove it is indeed an optimal solution.
2. Suppose the equation
$$a_1c_1 + a_2c_2 + \cdots + a_nc_n = 0$$
can only be satisfied by $a_i = 0$ for all $i = 1, \ldots, n$. Prove or disprove: the solution obtained in 1 is unique.

Problem 8 (12 pts)**.** Consider the following problem:

$$\begin{aligned} \text{maximize: } & Z = 3x_1 + 3x_2 + 2x_3 \\ \text{subject to: } & x_1 - x_2 + 4x_3 \leq 4 \\ & x_1 \leq 8 \\ & x_2 \leq 12 \\ & x_3 \leq 6 \end{aligned}$$

and $x_1 \geq 0$, $x_2 \geq 0$, and $x_3 \geq 0$.

Use the upper bound technique manually to solve this problem.

Problem 9 (12 pts)**.** Consider the following problem:

$$\begin{aligned} \text{maximize: } & Z = 2x_1 + 4x_2 + 3x_3 \\ \text{subject to: } & x_1 + 3x_2 + 2x_3 = 4 \\ & x_1 + 5x_2 \geq 10 \end{aligned}$$

and $x_1 \geq 0$, $x_2 \geq 0$, and $x_3 \geq 0$.

Use two phase method *or* Big M to derive an optimal solution, if there exists one. (Do not do both!)