Math 301: Homework 3

Due by email to `mtait@cmu.edu` Wednesday September 20 at noon

1. Recall that the n'th harmonic number is given by $H_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n}$.

   (a) Show that the generating function for H_n is

   $$\sum_{n \geq 0} H_n x^n = \frac{-\log(1-x)}{1-x}.$$

   (b) What is the generating function for $\sum_{k=1}^{n} H_k$?

2. Define a sequence $\{s_n\}$ by the recurrence $s_n = 3s_{n-1} + 2s_{n-2}$ where $s_1 = 1$ and $s_0 = 0$. Using generating functions, give an explicit formula for s_n.

3. Determine the number of k-element subsets of $[n]$ such that the ith largest element of the subset is congruent to i mod 2.

4. (a) Determine the generating function for the number of binary strings of length n that do not contain the string 0000.

   (b) Determine the generating function for the number of binary strings of length n that do not contain the string 0011.