Math 301: Homework 2

Due by email to `mtait@cmu.edu` Wednesday September 13 at noon

1. Let $\pi(n)$ denote the number of primes less than or equal to n. Show that $\pi(n) = O(n/\log n)$. (Hint: first show that for m a natural number, the product of the primes between m and $2m$ is bounded above by $\binom{2m}{m}$).

2. How many natural numbers are there up to 10^{30} that are either perfect squares, perfect cubes, or perfect fifth powers?

3. n married couples sit at a long table (of length n with 2 sides). How many ways can they sit so that no couple sits across from each other? How many ways can they sit such that exactly k couples sit across from each other?

4. Let n be large and choose two numbers from $[n]$ uniformly at random. What is the probability that they are coprime?