Name: _____ PID: _____

TA: _____ Sec. No: _____ Sec. Time: _____

Math 10B.
Midterm Exam 2
February 22, 2011.

Turn off and put away your cell phone.
You may use any type of calculator, but no other electronic devices during this exam.
You may use one page of notes, but no books or other assistance during this exam.
Read each question carefully, and answer each question completely.
Show all of your work; no credit will be given for unsupported answers.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

$\int_0^1 u + (u-1)^5$

#	Points	Score
1	6	
2	6	
3	6	
4	6	
Σ	24	

1. (6 points) Evaluate the integral $\int \frac{x+2}{(x-1)(x-4)}\,dx$ using partial fractions.

$$\frac{x+2}{(x-1)(x-4)} = \frac{A}{x-1} + \frac{B}{x-4} \qquad x+2 = A(x-4) + B(x-1)$$

$x = 1$: $A = 1$

$x = 4$: $B = 2$

$$\int \frac{x+2}{(x-1)(x-4)}\,dx = \int \frac{1}{x-1} + \frac{2}{x-4}\,dx$$

$$= \boxed{\ln|x-1| + 2\ln|x-4| + C}$$

2. (6 points) Evaluate the following integrals.

(a) $\int x^2\sqrt{x+4}\,dx$ $= \int (u-4)^2 u^{1/2}\,du$

$u = x+4$

$\frac{du}{dx} = 1$

$x = u-4$

$= \int (u^2 - 8u + 16)u^{1/2}\,du$

$= \int u^{5/2} - 8u^{3/2} + 16u^{1/2}\,du$

$= \frac{u^{7/2}}{7/2} - \frac{8u^{5/2}}{5/2} + \frac{16u^{3/2}}{3/2} + C$

$= \boxed{\frac{(x+4)^{7/2}}{7/2} - \frac{8(x+4)^{5/2}}{5/2} + \frac{16(x+4)^{3/2}}{3/2} + C}$

(b) $\int x^3 \ln(x)\,dx$

$u = \ln x \qquad v' = x^3$

$u' = \frac{1}{x} \qquad v = \frac{x^4}{4}$

$= \frac{(\ln x)x^4}{4} - \int \frac{x^3}{4}\,dx$

$= \boxed{\frac{(\ln x)x^4}{4} - \frac{x^4}{16} + C}$

3. (6 points) Consider the integral $\int_0^4 (x^2+1)\,dx$.

(a) Find $LEFT(2)$ and $RIGHT(2)$ for the integral.

(b) Find $MID(2)$ and $TRAP(2)$ for the integral.

SKIP

(c) Explain why $MID(2)$ is an underestimate and $TRAP(2)$ is an overestimate for the integral.

4. (6 points) Determine if the improper integral $\int_4^8 \frac{1}{\sqrt{x-4}}\,dx$ converges or diverges. If it converges, find its value.

$$\int_4^8 \frac{1}{\sqrt{x-4}}\,dx = \lim_{r\to 4^+} \int_r^8 \frac{dx}{\sqrt{x-4}}$$

$$= \lim_{r\to 4^+} 2\sqrt{x-4}\,\Big|_r^8$$

$$= \lim_{r\to 4^+} 2\sqrt{4} - 2\sqrt{r-4} \quad (\sqrt{r-4} \to 0)$$

$$= 4$$

$$\int_4^8 \frac{1}{\sqrt{x-4}}\,dx = 4 \quad \text{(converges)}$$