MATH 10B FALL 2012 Version 2.

1. Let $f(x) = \int_0^{x} \sin(e^t)\, dt$, then

$$\frac{d}{dx}\int_0^{x^2+1} \sin(e^t)\, dt = \frac{d}{dx} f(x^2+1) = f'(x^2+1)\cdot(x^2+1)'$$

$$= 2x \cdot \sin(e^{x^2+1})$$

2. (a) Intergration by ~~subst~~ parts, with $u = x$, $du = dx$

$dv = e^{-2x}dx$, $v = -\frac{1}{2}e^{-2x}$

$$\int x e^{-2x}\, dx = \int u\, dv = uv - \int v\, du = -\frac{1}{2}x e^{-2x} - \int -\frac{1}{2}e^{-2x}\, dx$$

$$= -\frac{1}{2}x e^{-2x} - \frac{1}{4}e^{-2x} + C$$

(b) Integration by Substitution, ~~with $u = (4-x^2)$, $du = -2x\,dx$~~

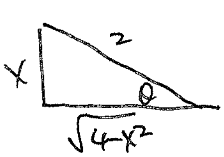
$x = \cancel{2\cos}\ 2\sin\theta$, $dx = 2\cos\theta\, d\theta$

$$\int \frac{1}{(4-x^2)^{3/2}}\, dx = \int \frac{1}{(4\cos^2\theta)^{3/2}} \cdot 2\cos\theta\, d\theta$$

$$= \frac{1}{4}\int \frac{1}{\cos^2\theta}\, d\theta$$

$$= \frac{1}{4}\int \sec^2\theta\, d\theta$$

$$= \frac{1}{4}\tan\theta + C = \frac{x}{4\sqrt{4-x^2}} + c$$



~~$\frac{1}{4}$~~

$\tan\theta = \frac{x}{\sqrt{4-x^2}}$

3. Integration by substitution :

$u = \ln x \qquad du = \frac{1}{x}\,dx$

$$\int \frac{1}{x(\ln x)^2}\,dx = \int \frac{1}{u^2}\,du = -\frac{1}{u} + C = -\frac{1}{\ln x} + C$$

$$\int_e^5 \frac{1}{x(\ln x)^2}\,dx = -\frac{1}{\ln x}\Big|_e^5 = -\frac{1}{\ln 5} + \frac{1}{\ln e}$$

$$= -\frac{1}{\ln 5} + 1$$

4 $$\frac{x+1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

$$= \frac{Ax(x-1) + B(x-1) + Cx^2}{x^2(x-1)}$$

$$= \frac{(A+C)x^2 + (B-A)x - B}{x^2(x-1)}$$

Hence, $A+C = 0$, $B-A = 1$, $B = -1$

$\Rightarrow$ $A = -2$, $B = -1$ $C = 2$, $\frac{x+1}{x^2(x-1)} = -\frac{2}{x} - \frac{1}{x^2} + \frac{2}{x-1}$

$$\int \frac{x+1}{x^2(x-1)}\,dx = \int -\frac{2}{x} - \frac{1}{x^2} + \frac{2}{x-1}\,dx = -2\ln|x| + \frac{1}{x} + 2\ln|x-1| + C$$

5 $$\int \frac{1}{\sqrt{x-2}}\,dx \underset{du=dx}{\overset{u=x-2}{=}} \int u^{-1/2}\,du = 2u^{1/2} + C = 2\sqrt{x-2} + C$$

$$\int_2^5 \frac{1}{\sqrt{x-2}}\,dx = 2\sqrt{5-2} - \lim_{x\to 2} 2\sqrt{x-2}$$

$$= 2\sqrt{3} - 0 = 2\sqrt{3}$$