

Midterm 1 Solutions

1. $x^2 = x + 2$ $x^2 - x - 2 = 0$ $x = -1, 2$

$$\text{Area} = \int_{-1}^2 x + 2 - x^2 = \left. \frac{x^2}{2} + 2x - \frac{x^3}{3} \right|_{-1}^2$$

$$= \left(2 + 4 - \frac{8}{3} \right) - \left(\frac{1}{2} - 2 + \frac{1}{3} \right)$$

2.

a. $\Delta \text{Change in position} = \int_0^{2\pi} \sin t \, dt$

$$= -\cos(2\pi) - (-\cos(0)) = 0$$

b. $\text{Distance} = \int_0^{2\pi} |\sin t| \, dt$

$$= \int_0^{\pi} \sin t - \int_{\pi}^{2\pi} \sin t \, dt$$

$$= \boxed{4}$$

3. a. 1

b. $1 + \frac{\pi}{2} - 1 = \frac{\pi}{2}$

c. $1 + 1 + \frac{\pi}{2} - 1 - 2$

d. $\frac{1}{6} \int_{-3}^3 g(x) dx = \boxed{\frac{\frac{\pi}{2} - 1}{6}}$

4. $\int_{-3}^2 f(x) dx = \int_{-3}^{-2} f(x) dx + \int_{-2}^2 f(x) dx$
 $= \boxed{12 + 0}$

b. $\int_{-2}^0 f(x) dx = 4$

$$\int_{-3}^0 f(x) dx = \int_{-3}^{-2} f(x) dx + \int_{-2}^0 f(x) dx = \boxed{6}$$

c. $F(2) - F(0) = \int_0^2 f(x) dx = -4$

$\boxed{F(2) = 1}$

d. $F(6) - F(-6) = 0$

$\boxed{F(6) = 10}$

5. increasing $\Rightarrow 2 \leq h(x) \leq 10$ on $0 \leq x \leq 8$

$$\boxed{16 = \int_0^8 2 dx \leq \int_0^8 f(x) dx \leq \int_0^8 10 dx = 80}$$