Name: _____ **PID:**_____

Circle your section: A01 (11am-12pm) or A02 (12pm-1pm)

MATH 10B: MIDTERM EXAM 2

July 26th, 2012

Do not turn the page until instructed to begin.

Turn off and put away your cell phone.

No calculators or any other devices are allowed.
You may use one 8.5×11 page of handwritten notes, but no other assistance.
Read each question carefully, answer each question completely, & show all of your work.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.
Good luck!

#	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
E.C.	3	
Σ	50	

1. Take the following indefinite integrals.

   (a) (5 points) $\displaystyle\int \frac{1}{(2-x)^2}\,dx.$

   (b) (5 points) $\displaystyle\int \frac{\ln(x)}{x^2}\,dx.$

2. Also take these indefinite integrals.

(a) (5 points) $\displaystyle\int \frac{2}{\sqrt{4-x^2}}\,dx.$

(b) (5 points) $\displaystyle\int \frac{2x}{\sqrt{4-x^2}}\,dx.$

3. Here are more indefinite integrals. Take them as well.

(a) (5 points) $\displaystyle\int \frac{x-2}{x^2(x+2)}\,dx.$

(b) (5 points) $\displaystyle\int \sin^2(x)\,\cos(x)\,dx.$

4. (10 points) You work for NASA and the current mission is to send a rover to explore a recently discovered planet in another galaxy. Your job is to determine the acceleration due to gravity for this planet. Once on the planet's surface, the rover drops a weight from a height of 2 meters and it measures how long it takes the weight to hit the ground. The rover determines this time is 1 second. What is the acceleration due to gravity on this planet?

   (Your answer is assumed to be in units of m/s^2)

5. For each of the following integrals, if it converges, write the number it converges to. Otherwise, you may simply write "diverges" as your answer.

(a) (5 points) $\displaystyle\int_1^\infty \frac{1}{x\sqrt{x}}\,dx.$

(b) (5 points) $\displaystyle\int_0^5 \frac{1}{(x-1)^2}\,dx.$

Extra Credit: (3 points) Take one more indefinite integral:

$$\int e^{\sqrt{x}}\, dx$$

TABLE OF INTEGRALS

BASIC FUNCTIONS

1. $\displaystyle\int x^n\,dx = \frac{1}{n+1}\,x^{n+1} + C \qquad (\text{if } n \neq 1)$
2. $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$
3. $\displaystyle\int a^x\,dx = \frac{1}{\ln(a)}\,a^x + C \qquad (\text{if } a > 0)$
4. $\displaystyle\int \ln(x)\,dx = x\,\ln(x) - x + C$
5. $\displaystyle\int \sin(x)\,dx = -\cos(x) + C$
6. $\displaystyle\int \cos(x)\,dx = \sin(x) + C$
7. $\displaystyle\int \tan(x)\,dx = -\ln|\cos(x)| + C$

PRODUCTS OF e^x, $\cos(x)$, $\sin(x)$

8. $\displaystyle\int e^{ax}\sin(bx)\,dx = \frac{1}{a^2+b^2}\,e^{ax}\left[a\sin(bx) - b\cos(bx)\right] + C$
9. $\displaystyle\int e^{ax}\cos(bx)\,dx = \frac{1}{a^2+b^2}\,e^{ax}\left[a\cos(bx) + b\sin(bx)\right] + C$
10. $\displaystyle\int \sin(ax)\sin(bx)\,dx = \frac{1}{b^2-a^2}\left[a\cos(ax)\sin(bx) - b\sin(ax)\cos(bx)\right] + C \quad (\text{if } a \neq b)$
11. $\displaystyle\int \cos(ax)\cos(bx)\,dx = \frac{1}{b^2-a^2}\left[b\cos(ax)\sin(bx) - a\sin(ax)\cos(bx)\right] + C \quad (\text{if } a \neq b)$
12. $\displaystyle\int \sin(ax)\cos(bx)\,dx = \frac{1}{b^2-a^2}\left[b\sin(ax)\sin(bx) + a\cos(ax)\cos(bx)\right] + C \quad (\text{if } a \neq b)$

PRODUCT OF POLYNOMIAL $p(x)$ WITH $\ln(x)$, e^x, $\cos(x)$, $\sin(x)$

13. $\displaystyle\int x^n\ln(x)\,dx = \frac{1}{n+1}\,x^{n+1}\ln(x) - \frac{1}{(n+1)^2}x^{n+1} + C \qquad (\text{if } n \neq -1)$
14. $\displaystyle\int p(x)e^{ax}\,dx = \frac{1}{a}p(x)e^{ax} - \frac{1}{a}\int p'(x)\,e^{ax}\,dx$
$\displaystyle= \frac{1}{a}p(x)e^{ax} - \frac{1}{a^2}p'(x)e^{ax} + \frac{1}{a^3}p''(x)\,e^{ax} - \cdots \qquad (+-+-+-\ldots)$
15. $\displaystyle\int p(x)\sin(ax)\,dx = -\frac{1}{a}p(x)\cos(ax) + \frac{1}{a}\int p'(x)\cos(ax)\,dx$
$\displaystyle= -\frac{1}{a}p(x)\cos(ax) + \frac{1}{a^2}p'(x)\sin(ax) + \frac{1}{a^3}p''(x)\cos(ax) - \cdots \quad (-++--++\ldots)$
16. $\displaystyle\int p(x)\cos(ax)\,dx = \frac{1}{a}p(x)\sin(ax) - \frac{1}{a}\int p'(x)\sin(ax)\,dx$
$\displaystyle= \frac{1}{a}p(x)\sin(ax) + \frac{1}{a^2}p'(x)\cos(ax) - \frac{1}{a^3}p''(x)\sin(ax) - \cdots \quad (++--++\ldots)$

INTEGER POWERS OF $\sin(x)$, $\cos(x)$

17. $\displaystyle\int \sin^n(x)\,dx = -\frac{1}{n}\sin^{n-1}(x)\,\cos(x) + \frac{n-1}{n}\int \sin^{n-2}(x)\,dx + C \qquad (\text{if } n > 0)$

18. $\displaystyle\int \cos^n(x)\,dx = \frac{1}{n}\cos^{n-1}(x)\,\sin(x) + \frac{n-1}{n}\int \cos^{n-2}(x)\,dx + C \qquad (\text{if } n > 0)$

19. $\displaystyle\int \frac{1}{\sin^m(x)}\,dx = \frac{-1}{m-1}\,\frac{\cos(x)}{\sin^{m-1}(x)} + \frac{m-2}{m-1}\int \frac{1}{\sin^{m-2}(x)}\,dx \qquad (\text{if } m > 1)$

20. $\displaystyle\int \frac{1}{\sin(x)}\,dx = \frac{1}{2}\ln\left|\frac{\cos(x)-1}{\cos(x)+1}\right| + C$

21. $\displaystyle\int \frac{1}{\cos^m(x)}\,dx = \frac{1}{m-1}\,\frac{\sin(x)}{\cos^{m-1}(x)} + \frac{m-2}{m-1}\int \frac{1}{\cos^{m-2}(x)}\,dx \qquad (\text{if } m > 1)$

22. $\displaystyle\int \frac{1}{\cos(x)}\,dx = \frac{1}{2}\ln\left|\frac{\sin(x)+1}{\cos(x)-1}\right| + C$

23. $\displaystyle\int \sin^m(x)\cos^n(x)\,dx$

If m is odd, let $w = \cos(x)$. If n is odd, let $w = \sin(x)$. If both m and n are even and nonnegative, convert all to $\sin(x)$ or all to $\cos(x)$ (using $\cos^2(x) + \sin^2(x) = 1$), and use 17 or 18. If m and n are even and one of them is negative, convert to whichever function is in the denominator and use 19 or 21. If both m and n are even and negative, substitute $w = \tan(x)$.

QUADRATIC IN THE DENOMINATOR

24. $\displaystyle\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$

25. $\displaystyle\int \frac{bx+c}{x^2+a^2}\,dx = \frac{b}{2}\ln|x^2+a^2| + \frac{c}{a}\arctan\left(\frac{x}{a}\right) + C \qquad (\text{if } a \neq 0)$

26. $\displaystyle\int \frac{1}{(x-a)(x-b)}\,dx = \frac{1}{a-b}(\ln|x-a| - \ln|x-b|) + C \qquad (\text{if } a \neq b)$

27. $\displaystyle\int \frac{cx+d}{(x-a)(x-b)}\,dx = \frac{1}{a-b}[(ac+d)\ln|x-a| - (bc+d)\ln|x-b|] + C \qquad (\text{if } a \neq b)$

INTEGRANDS INVOLVING $\sqrt{a^2+x^2}$, $\sqrt{a^2-x^2}$, $\sqrt{x^2-a^2}$, $\quad a > 0$

28. $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right) + C$

29. $\displaystyle\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

30. $\displaystyle\int \sqrt{a^2 \pm x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2 \pm x^2} + a^2\int \frac{1}{\sqrt{a^2 \pm x^2}}\,dx\right) + C$

31. $\displaystyle\int \sqrt{x^2-a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2-a^2} - a^2\int \frac{1}{\sqrt{x^2-a^2}}\,dx\right) + C$