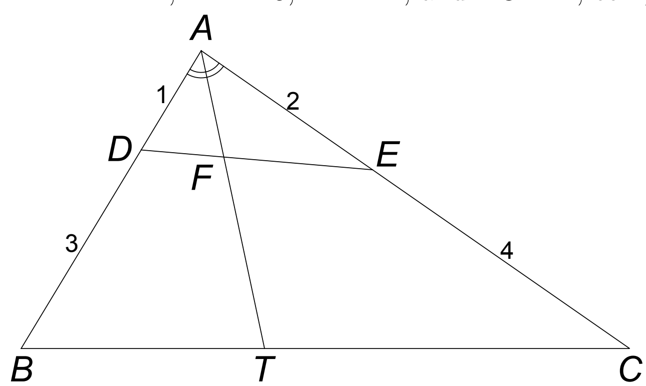
Geometry	Misha Lavrov

Triangle Geometry

Western PA ARML Practice *November 1, 2015*

1 ARML problems

1. (ARML 1980) In $\triangle ABC$, the angle bisector AI divides the median BM into two segments of length 200 and 300, and AI divides BC into two segments of length 660 and x. Find the largest possible value of x.
2. (ARML 1992) In $\triangle ABC$, points D and E are on AB and AC, and the angle bisector AT intersects DE at F. If $AD = 1$, $DB = 3$, $AE = 2$, and $EC = 4$, compute the ratio $AF : AT$.



3. (ARML 1992) Points P, Q, and R are the midpoints of the medians of $\triangle ABC$. If the area of $\triangle ABC$ is 1024, compute the area of $\triangle PQR$.

2 Properties of angle bisectors and incircles

1. Prove the Angle Bisector Theorem: if AD bisects $\angle A$, then $AB : AC = BD : CD$.
2. In $\triangle ABC$, the altitude AP and median AQ trisect $\angle A$. Find the angles of $\triangle ABC$.
3. In the convex quadrilateral $ABCD$, the inradii of $\triangle ABC$, $\triangle BCD$, $\triangle CDA$, and $\triangle DAB$ are equal. Prove that $AC = BD$.

3 Properties of medians

1. In $\triangle ABC$, let the medians AM and BN intersect at X. Find the ratio $AX : XM$.
2. In $\triangle ABC$, let the medians AM and BN intersect at X, and let CX meet AB at P. Prove that $AP = BP$, which shows that the three medians of $\triangle ABC$ meet at a common point.

3. Show that if the medians of $\triangle ABC$ intersect at X, then the areas of $\triangle ABX$, $\triangle ACX$, and $\triangle BCX$ are equal.
4. The medians of a triangle have lengths 9, 12, and 15. Find the area.