21-121 **IM/Econ Calculus I** **Fall 2000**

Review Problems for Test 2

1. Find $f'(x)$ for each of the following.

   (a) $f(x) = \sqrt{x^4+1}$

   (b) $f(x) = 3\sin(x^2+1) + 2e^{-x}$

   (c) $f(x) = x^3 \ln(1+x^4)$

   (d) $f(x) = \cos^{-1}(\sqrt{x})$

   (e) $f(x) = \dfrac{e^{\sin x}}{1+e^{\cos x}}$

   (f) $f(x) = 2\ln(e^x + e^{-x})$

   (g) $f(x) = \sin^{-1}(2x) + \tan^{-1}(3x)$

2. Solve for x.

   (a) $e^{3x} = 6e^{-2x}$

   (b) $\ln x + \ln(x-1) = \ln 6$

3. Find the tangent line to the curve $xy^4 + y + x^3 = 2$ at the point (1,1)

4. Find $f'(x),\ f''(x),\ f'''(x)$ for $f(x) = \sqrt{1-3x}$.

5. Find $\dfrac{dy}{dx}$ for each of the following.

   (a) $y = (1+x^2)^{\cos x}$

   (b) $y = (1+\sqrt{x})^{\ln x}$

6. Find $\dfrac{d^2y}{dx^2}$ if $y^3 + xy = 2$.

7. Suppose that the demand for a manufacturer's product satisfies

$$p^3 + pq + 10p = 2000$$

   where p is the price per unit and q is the number of units.

   (a) Use implicit differentiation to find $\dfrac{dp}{dq}$.

   (b) Find the marginal revenue when $p = 10,\ q = 90$.

8. Suppose that f and F are differentiable functions satisfying $F(x) > 0$ for all x; $f'(x) = F(x)$ and $F'(x) = f(x)$ for all x; $(F(x))^2 - (f(x))^2 = 1$ for all x, and f is one to one. Let g denote the inverse function of f, i.e. $y = g(x)$ if and only if $x = f(y)$. Find an explicit formula for $g'(x)$.

9. A CMU business student and a CMU engineering student are talking with each other at the same spot on campus. At the instant when the conversation ends, the business students starts walking north at 3 meters per second and the engineering student starts walking east at 2 meters per second. How fast is the distance between the students changing at the instant precisely 30 seconds after the conversation has ended.

10. A spherical balloon is being inflated at a constant rate of 16 ft^3 per minute. If R denotes the radius of the balloon, find $\dfrac{d^2R}{dt^2}$ at the instant when $R = 2$ ft. (Here t is time.)

11. Assume that f is differentiable and satisfies $f(6) = 11$ and $f'(6) = 3$. Find an approximate value for $f(5.9)$.

12. Find the absolute maximum and the absolute minimum of $f(x) = x^{2/3} - \frac{2}{3}x + 1$ on $[-1, 1.5]$.