21-121 **IM/Econ Calculus I** **Fall 2000**

Review Problems for Test 1

1. Find the indicated limit or explain why it does not exist.

$$\text{(a)}\ \lim_{x\to 2}\frac{x^3+x-2}{x-1} \qquad \text{(b)}\ \lim_{x\to 1}\frac{x^2-3x+2}{x^3-1}$$

$$\text{(c)}\ \lim_{x\to 3^+}\frac{|3-x|}{9-x^2} \qquad \text{(d)}\ \lim_{x\to 3}\frac{|3-x|}{9-x^2}$$

2. Consider the function f defined by

$$f(x)=\begin{cases} -1 & x<0 \\ x^2 & 0\le x<1 \\ 2 & x=1 \\ x & 1<x<2 \\ 3-x & 2\le x \end{cases}$$

   (a) Sketch the graph of $y=f(x)$.

   (b) Evaluate $\lim_{x\to 1} f(x)$ or explain why it does not exist.

   (c) Where is f continuous?

3. Use the definition of derivative to find $f'(x)$ for each of the following:

   (a) $f(x)=1+3x-x^2$ (b) $f(x)=\dfrac{2}{x+1}$

   (c) $f(x)=\sqrt{x-1}$

4. Show that the equation

$$x^{10}+\frac{1}{\sqrt{x+3}}-10x+2=0$$

   has at least one (real) root.

5. Find the tangent line to the curve at the point P.

   (a) $y=\dfrac{x}{x^2+1}$ $\qquad P=\left(1,\dfrac{1}{2}\right).$

   (b) $y=x^4-3x^2+5$ $\qquad P=(-1,3)$

6. Find $f'(x)$ for each of the following:

   (a) $f(x) = \left(x^3 - \frac{1}{x}\right)(e^{3x} - 2e^{5x})$

   (b) $f(x) = \dfrac{4}{x^5 - 2x + 7}$

   (c) $f(x) = \sqrt{x}e^{-x}$

   (d) $f(x) = (x^2+1)^3(x^2-1)$

   (e) $f(x) = \sqrt{x} + \frac{1}{x} + \frac{1}{x^2}$

7. Suppose that the demand for a monopolists product is given by $p = 2 + \dfrac{200}{\sqrt{q}}$, where p is the price per unit and q is the number of units, and that the average cost function is given by $\overline{c} = 3 + \dfrac{100}{q}$.

   (a) Find the marginal revenue function.

   (b) Find the marginal cost function.

   (c) Find an expression for the profit P as a function of q.

   (d) Find all values of q at which $\dfrac{dP}{dq} = 0$.

8. Suppose that the demand for a monopolists product is given by $p = F(q)$ where F is a function satisfying $F(100) = 3$ and $F'(100) = -.01$. Find the marginal revenue when $q = 100$.

9. Find a line that passes through the point $\left(1, \frac{11}{4}\right)$ and is tangent to $y = x^2 + 2x$ somewhere.

10. Evaluate the following limit or explain why it does not exist:

$$\lim_{x \to 0^+} \left(\frac{1}{\sqrt{x^2+x}} - \frac{1}{\sqrt{x}} \right).$$

11. Assume that $f : (0, \infty) \to \mathbb{R}$ satisfies $f(s) - f(t) = f(\frac{s}{t})$ for all $s,\ t > 0$ and $\lim\limits_{h \to 0} \dfrac{f(1+h)}{h} = 3$. Show that f is differentiable on $(0, \infty)$ and find a formula for $f'(x)$.