Review Problems for Test 1

1. Let $\vec{a} = < 1, 3, -1 >$, $\vec{b} = < 0, 4, 2 >$, $\vec{c} = < 5, -1, 3 >$. Compute each of the following:

   (a) $\left|\vec{a}\right|$

   (b) $\vec{a} \cdot \vec{c}$

   (c) the angle between $\vec{a}$ and $\vec{c}$

   (d) the vector projection of $\vec{b}$ onto $\vec{a}$

   (e) the direction cosines of $\vec{c}$

   (f) $\vec{a} \cdot (\vec{b} \times \vec{c})$

2. (a) Write an equation for the plane which passes through $(3, -6, 8)$ and is normal to $< 7, -2, -3 >$.

   (b) Write the symmetric and parametric equations of the line which passes through $(1, 3, -5)$ and is parallel to $< 7, -2, -3 >$.

3. Find the line of intersection of the planes $x + y + z = 4$ and $2x - y + 5z = -1$.

4. Find a number d such that the distance from the point $P = (4, 3, 0)$ to the plane $x - 2y + z = d$ is 3.

5. Find the distance from the point $(1, 0, -1)$ to the line $x = t,\ y = -1 - t,\ z = 1 + 2t$.

6. Let $\vec{a} = < 2, 0, -1 >$ and $\vec{b} = < 1, 2, 0 >$. Write $\vec{b}$ as the sum of a vector $\vec{b}_1$ parallel to $\vec{a}$ and a vector $\vec{b}_2$ perpendicular to $\vec{a}$.

7. Find the points of intersection of the line $\frac{x}{-2} = \frac{y-1}{3} = \frac{z}{\sqrt{3}}$ and the sphere $x^2 + 3x + y^2 + z^2 = 2$.

8. Find the plane that contains the line $x = y = z$ and the point $P_0 = (1, 2, 3)$.

9. Given that $\vec{a}$ and $\vec{b}$ are vectors with $\left|\vec{a}\right| = 1$ and $\left|\vec{a} - \vec{b}\right| = \left|\vec{b}\right|$, find $\vec{a} \cdot \vec{b}$.

10. Let $\vec{a} = < 1, 1, 1 >$, $\vec{b} = < 2, -3, 1 >$ and $\vec{c} = < 1, 0, 2 >$. Find a vector $\vec{v}$ satisfying

$$\vec{a} \cdot \vec{v} = \vec{b} \cdot \vec{v} = 0$$
$$\vec{c} \cdot \vec{v} = 2.$$

11. Let $\vec{v}$ be a vector satisfying $\left|\vec{v} \times \vec{i}\right| = A, \left|\vec{v} \times \vec{j}\right| = B, \left|\vec{v} \times \vec{k}\right| = C$ Express $\left|\vec{v}\right|$ in terms of A, B, C.

12. (a) A space curve is described by a vector function $\vec{r}\,(t)$ satisfying

$$\vec{r'}\,(t) = < e^t \cos 2t, e^t \sin 2t, e^t > .$$

Find the unit tangent vector $\vec{T}\,(t)$, the principal unit normal vector $\vec{N}\,(t)$, the binormal vector $\vec{B}(t)$, and the curvature $\kappa(t)$.

(b) The torsion τ of a space curve is defined by $\tau = \; -\vec{N} \cdot \dfrac{d\vec{B}}{dS}$.
Find the torsion of the curve from part (a) when $t = 0$.

13. Find the length of the curve described by

$$\vec{r}\,(t) = < \sin 3t, \cos 3t, 2t^{3/2} >$$

for $1 \leq t \leq 3$.

14. Find the limit, or show that the limit does not exist.

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 + y^2 + 2x^4}{x^2 + y^2}$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3}{xy^2 + y^3}$

15. Suppose that $f\,(x, y) = \left(y - x^2\right)^2$. Sketch the level curves of f for $c = 0$ and $c = 1$.

16. Suppose that $f\,(x, y) = \sin\,(x + 3y) + x^2y^3 + 2y^2$. Compute f_x, f_y, f_{xx}, f_{xy}, and f_{yy}.