Department of Mathematical Sciences
Carnegie Mellon University

21-476 **Ordinary Differential Equations** **Fall 2003**

Assignment 7

Due on Monday, December 1

1. Assume that $g : \mathbb{R}^n \to \mathbb{R}^n$ is continuous, has the uniqueness property, and satisfies $g(0) = 0$, and consider the autonomous system

   $$\dot{x} = g(x). \tag{1}$$

   Let a continuously differentiable function $V : \mathbb{R}^n \to \mathbb{R}$ be given and define $\dot{V} : \mathbb{R}^n \to \mathbb{R}$ by $\dot{V}(z) = \nabla V(z) \cdot g(z)$. Show that if $\dot{V}(z) < 0$ for all $z \in \mathbb{R}^n \backslash \{0\}$ then (1) has no nonconstant periodic solutions.

2. Give an example of a planar autonomous system

   $$\dot{x} = g(x),$$

   where $g : \mathbb{R}^2 \to \mathbb{R}^2$ is smooth, and a point $p \in \mathbb{R}^2$ such that $\gamma^+(p)$ is bounded and $\omega(p)$ contains more than one critical point; **or else** show that such an example is impossible.

   In problems 3 and 4 below determine as much as you can about solutions. Pay particular attention to issues concerning periodic orbits.

3. $$\begin{aligned} \dot{x}_1 &= x_2 - x_1^3 \\ \dot{x}_2 &= -x_2 - x_2^3 + x_1^3 \end{aligned}$$

4. $$\begin{aligned} \dot{x}_1 &= x_1 - x_2 - x_1^3 \\ \dot{x}_2 &= x_1 + x_2 - x_2^3 \end{aligned}$$