Department of Mathematical Sciences
Carnegie Mellon University

21-476 Ordinary Differential Equations Fall 2003

Assignment 5

Due on Friday, November 7

1. Let $G : \mathbb{R}^n \to \mathbb{R}$ be given and assume that G is twice continuously differentiable, $G(z) \to \infty$ as $||z|| \to \infty$, and that there is exactly one $x^* \in \mathbb{R}^n$ such that $\nabla G(x^*) = 0$. Discuss the behavior as $t \to \infty$ of solutions of

$$\dot{x} = -\nabla G(x). \tag{2}$$

2. Let $\alpha, g, l > 0$ be given and consider the system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\tfrac{g}{l} \sin x_1 - \alpha x_2. \end{aligned}$$

   Show that for every $p \in \mathbb{R}^2$, $\gamma^+(p)$ is bounded.

3. Determine as much as you can about the stability of $(0, 0)$ by studying a suitable Liapunov function

   (a) $\dot{x}_1 = -x_1^3 + x_2$

   $\dot{x}_2 = x_1^2\ x_2 + x_1$

   (b) $\dot{x}_1 = 3x_1^2\ x_2 - x_1^5$

   $\dot{x}_2 = -x_1^3\ - x_2^3$

   (c) $\dot{x}_1 = -x_1^3 + 2x_2^3$

   $\dot{x}_2 = -2x_1\ x_2^2$