Department of Mathematical Sciences
Carnegie Mellon University

21-476 **Ordinary Differential Equations** **Fall 2003**

Assignment 4

Due on Friday, October 10

1. Let $(\alpha, \beta) \in \mathbb{R}^2$ be given. Let x be the unique noncontinuable solution of

$$\begin{cases} \dot{x}_1(t) = e^{-t}x_1(t) + e^{-2t}x_2(t) \\ \dot{x}_2(t) = -e^{-2t}x_1(t) + e^{-t}x_2(t) \\ x_1(0) = \alpha, \;\; x_2(0) = \beta \end{cases}$$

and let y be the unique noncontinuable solution of

$$\begin{cases} \dot{y}_1(t) = e^{-t}y_1(t) + e^{2t}y_2(t) \\ \dot{y}_2(t) = -e^{2t}y_1(t) + e^{-t}y_2(t) \\ y_1(0) = \alpha, \;\; y_2(0) = \beta. \end{cases}$$

In Problem 2 of Assignment 3, you showed that $[0, \infty) \subset \mathrm{Dom}(x)$ and you studied the behavior as $t \to \infty$ of $||x(t)||_2$.

   (a) Show that $[0, \infty) \subset \mathrm{Dom}(y)$.

   (b) What can you say about the behavior of $||y(t)||_2$ as $t \to \infty$?

   (c) What, if anything, is the difference between the behavior of $x(t)$ as $t \to \infty$ and the behavior of $y(t)$ as $t \to \infty$?

2. Show that the system

$$\dot{x}_1(t) = x_2(t) - x_1(t) + \sin t$$

$$\dot{x}_2(t) = -x_1(t) + (1 - x_1(t)^2 - x_2(t)^2)x_2(t)$$

has a 2π-periodic solution.

3. Show that the system

$$\dot{x}_1(t) = 2 + \cos t + \left(1 + x_2(t)^2\right)\left(1 - x_1(t)^2\right)$$

$$\dot{x}_2(t) = 3 - \sin t + \left(2 + x_1(t)^2\right)\left(4 - x_2(t)^2\right)$$

has a 2π-period solution.

4. Draw the phase portrait for the autonomous system

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1 + x_1^3.$$

What can you deduce about the behavior of solutions form the phase portrait?

5. If you did not turn in a solution to Problem 4 on Assignment 3 or if you wish to revise your solution to that problem, you may do so now.