21-259 **Calculus in 3-D** **Spring 2003**

Answers to Review Problems for Test 2

1. (a) $\vec{\nabla} f(x,y,z) = \langle y, x+2yz, y^2 \rangle$

   (b) $\dfrac{-37}{\sqrt{29}}$

2. $\vec{\nabla} f(0,0,0) = \dfrac{-5}{\sqrt{41}} \langle 6, -1, 2 \rangle$

3. Tangent Plane: $-2(x+1) + 16(y-2) - (z-17) = 0$

   Normal Line: $\dfrac{x+1}{-2} = \dfrac{y-2}{16} = \dfrac{z-17}{-1}$

4. $\vec{\nabla} f(2,1) = \langle -6, 10 \rangle$

5. $(2,1,-2)$ and $(-2,-1,2)$

6. $3(rs)^2(r^2-s^2)^3 r - 6s\left[(rs)^3 + 2(r^2-s^2)^3\right](r^2-s^2)^2 - 12(2r+s)^2$

9. $4u\dfrac{d^2w}{du^2} + 4\dfrac{dw}{du}$

10. (a) relative min at $(-1,1)$

    (b) relative min at $(0,0)$; saddle points at $\left(\dfrac{1}{2}, -1\right)$ and $(2,2)$

11. Absolute min occurs at (2,1). Absolute max occurs at (0,4). $f(2,1) = -4$ and $f(0,4) = 9$

12. (a) Possible maxima and minima at $(0,1,-3)$, $(0,-1,3)$ $\left[\text{corresponding to } \lambda = \pm\dfrac{1}{2}\right]$

    and at $\left(\lambda, \dfrac{1}{2\lambda}, \dfrac{-3}{2\lambda}\right)$ with $\lambda = \pm\sqrt{\dfrac{10+\sqrt{90}}{2}}$ and $\lambda = \pm\sqrt{\dfrac{10-\sqrt{90}}{2}}$.

    (b) Possible maxima and minima at $\left(\dfrac{\sqrt{2}}{3}, \sqrt{2}\right)$, $\left(\dfrac{-\sqrt{2}}{3}, -\sqrt{2}\right)$, $\left(-\dfrac{\sqrt{2}}{3}, \sqrt{2}\right)$,

    $\left(\dfrac{\sqrt{2}}{3}, -\sqrt{2}\right)$,

13. (a) $\displaystyle\int_0^1 \int_{x^2}^{x} f(x,y)\,dy\,dx$

    (b) $\displaystyle\int_1^{e^4} \int_{\frac{1}{2}lny}^{2} f(x,y)\,dx\,dy$

    (c) $\displaystyle\int_0^4 \int_0^{y/2} f(x,y)\,dx\,dy$

14. (a) $\dfrac{2}{9}$

(b) $\dfrac{(2^{3/2} - 1)}{4}$