21-355 **Principles of Real Analysis I** **Fall 2004**

Assignment 0

Solutions to these problems do not need to be turned in.

1. Let $\mathbb{C}$ denote the complex numbers with the usual addition and multiplication. Show that there is no set $\mathbb{C}^+ \subset \mathbb{C}$ satisfying (i), (ii), (iii) below:

   (i) $\forall z, w \in \mathbb{C}^+, \quad z + w \in \mathbb{C}^+$ and $z \cdot w \in \mathbb{C}^+$;

   (ii) $\forall z \in \mathbb{C}^+, \quad -z \notin \mathbb{C}^+$;

   (iii) $\forall z \in \mathbb{C}, \quad z = 0$ or $z \in \mathbb{C}^+$ or $-z \in \mathbb{C}^+$.

In problems 2-10, prove the given assertion from basic principles.

2. $\forall x \in \mathbb{R}, \quad (-1) \cdot x = -x.$

3. $\forall x \in \mathbb{R}, \quad x \cdot 0 = 0.$

4. $\forall x, y \in \mathbb{R}, \quad (-x) \cdot y = x \cdot (-y) = -(xy).$

5. $\forall x, y \in \mathbb{R}, \quad (-x) \cdot (-y) = xy.$

6. $\forall x \in \mathbb{R}\backslash\{0\}, \quad x^2 > 0.$ (Here $x^2 = x \cdot x$.)

7. $\mathbb{N} \subset \mathbb{P}$

8. Let $x, y \in \mathbb{R}$ be given. If $xy = 0$ then $x = 0$ or $y = 0$.

9. Let $x, y, z \in \mathbb{R}$ be given. If $x < y$ and $y < z$ then $x < z$.

10. Let $a, b, c, d \in \mathbb{R}$ with $b \neq 0$ and $d \neq 0$ be given. Then $\left(\frac{a}{b}\right) + \left(\frac{c}{d}\right) = \frac{(ad) + (bc)}{(bd)}$.