**21-341** **Linear Algebra I** **Fall 2003**

## Supplementary Problems for Assignment 2

1. Let $P(\mathbb{R})$ denote the vector space of all real polynomials over the field $\mathbb{R}$. For each $n \in \mathbb{Z}^+$, let $P_n(\mathbb{R})$ denote the subset of $P(\mathbb{R})$ consisting of all polynomials of degree $\leq n$. Let $P_0(\mathbb{R}) = \{0\}$, the set consisting of the zero polynomial.

   (a) Let $S = \{f \in P_2(\mathbb{R}) : \int_0^1 f(x)dx = 0\}$. Show that $S$ is a subspace of $P(\mathbb{R})$ and find a basis for $S$.

   (b) Let $T = \{f \in P_4(\mathbb{R}) : f'(0) = f(1) = 0\}$. Show that $T$ is a subspace of $P(\mathbb{R})$ and find a basis for $T$.

2. Let $\mathcal{F}(\mathbb{R})$ be the vector space of all real-valued functions on $\mathbb{R}$ over the field $\mathbb{R}$. Define $f_1, f_2, f_3 \in \mathcal{F}(\mathbb{R})$ by $f_1(x) = x, \quad f_2(x) = e^x, \quad f_3(x) = \sin x$ forall $x \in \mathbb{R}$. Show that $f_1, f_2, f_3$ are linearly independent.

3. Let $\mathbb{F}$ be a field and $V, W$ be vector spaces over $\mathbb{F}$. Let $L : V \to W$ be a mapping (i.e. function) satisfying $L(u+v) = L(u)+L(v), \quad L(\lambda u) = \lambda L(u)$ for all $u, v \in V, \ \lambda \in \mathbb{F}$. Let $S = \{u \in V : L(u) = 0\}$. Show that $S$ is a subspace of $V$.

4. Let $\mathbb{F}$ be a field, $V$ be a vector space over $\mathbb{F}$ and $S_1, S_2$ be subspaces of $V$. Let $T = \{u + v : u \in S_1, \ v \in S_2\}$. Show that $T$ is a subspace of $V$.

5. Let $\mathbb{F} = \mathbb{Z}_5$ and $V = \mathbb{F}^3$, i.e. the set of all ordered 3-tuples from $\mathbb{F}$. Determine whether or not the vectors $< 1, 0, 0 >, \ < 1, 4, 1 >, \ < 4, 1, 4 >$ are linearly independent.

6. Let $\mathbb{F} = \mathbb{R}$ and $V = \mathbb{R}^3$. Determine whether or not the vectors $< 1, 0, 0 >$, $< 1, 4, 1 >, \ < 4, 1, 4 >$ are linearly independent.