21-241 : Matrix Algebra
Summer I 2014

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Office Hours:	
Monday:	4-5
Tuesday	2-3
Wednesday	4-5
Thursday	2-3
Friday	4-5
(or by appointment)	
Course Dates:	May 19 - June 27
Course Webpage	`http://math.cmu.edu/~wgunther/241/m14/`
Lecture Times:	Weekdays 10:30-11:50am
Lecture Place:	HH B131
Textbook:	*Linear Algebra: A Modern Introduction* by David Poole (and occasional supplemental material on website)

Introduction: This course is an introductory course in linear algebra. Linear algebra is, in simplest terms, a study of a particular kind of function between particular kinds of object. The applications of this field are immense. Historically, vectors found a place in physics and engineering as a way to describe physical phenomena (such as force). Nowadays, they span almost every scientific and technical field, including the field of computer science, and are often imagined in different ways than their original intent.

Let's delve in a little deeper into what objects and functions we're talking about. The objects are *vectors* in some *vector space*; these are objects that have particular algebraic properties. For most of the course, our vectors be vectors in the vector space $\mathbb{R}^n$. These will be formally defined in the course, but these are the vectors that you may be acquainted with from other courses, such as physics; they have a length and a point in some direction in space. Later in the course we will explore some more abstract vector spaces.

The functions that we are interested in are *linear functions* between two vector spaces. Linear functions 'respect' vector operations. Linear functions are mostly what you might expect from learning about lines in pre-calculus, except we impose an addition restriction that the line goes through the origin, i.e. preserves 0. These linear functions between vector spaces (which we will call *linear transformations*) can be realized (in most practical cases) as *matrices*. We will therefore reduce the study of a large, complicated set of functions to algebraic properties of matrices.

Prerequisites: This course is an introductory course. Therefore, there are no official prerequisites. But, a good grasp of algebra is necessary. A high school pre-calculus course is sufficient. If it turns out that everyone in the course has taken calculus, we will be able to have a richer set of examples.

The most difficult part of this course will be that it is a proof based course. This means that beyond learning the language, techniques, and applications of linear algebra, we will also present the theory and justify ourselves with formal arguments. For someone who has not taken a proof based course before, this may very different than you are used to. It is recommended that you have taken 21-127 (Concepts of Mathematics) before this course, but it is not required, nor is it necessary for a sufficiently dedicated student.

Learning Objectives (*under construction***):** By the end of the course, you are expected to have the ability to:

- System of Linear Equations
  - Identify a system of linear equations
  - Compute a corresponding augmented matrix for a system
  - Apply a relevant algorithm to obtain a solution set
  - Summarize the results of the algorithm.
  - Construct the solution set

  - Explain what can be determined about the underlying coefficient matrix when viewed as a linear transformation.
  - Explain what can be determined about the underlying column and row vectors of the matrix.
- Vectors of $\mathbb{R}^n$
  - Explain what a vector in $\mathbb{R}^n$ is.
  - Outline different operations that you can perform on these vectors.
  - Illustrate possible applications for talking about vectors in $\mathbb{R}^n$.
  - Sketch a geometric interpretation of vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$ and the geometric interpretation of different operations (e.g. addition).
  - Explain what linear dependence, span, dimension, and a basis are.
  - Interpret results about matrices and linear transformations to tell you results about sets of vectors.
- Matrix Algebra
  - Represent a system of linear equations as an augmented matrix
  - Perform matrix multiplication, and understand its significance in terms of the column vectors of the matrix.
  - Understand the connection between matrices and linear transformations.
  - Define the inverse and transpose of a matrix. Understand the algebraic significance.
  - Define various matrix parameters, such as size, rank, range, row space, null space, nullity
  - Calculate different parameters of a matrix, such as the size, rank, range, row space, null space, nullity
  - Understand, formulate, and prove different connections between matrix parameters (e.g. rank + nullity = number of columns)
- Eignevalues and Determinants
  - Define an eigenvalue, eigenvector, and spectrum of a matrix
  - Sketch a geometric interpretation of an eigenvalue and eigenvector in $\mathbb{R}^2$ and $\mathbb{R}^3$.
  - Calculate the determinant of a matrix
  - Understand, formulate, and prove various properties of determinants.
  - Apply properties of determinants to determine information about the matrix/linear transformation (e.g. invertability).
  - Calculate the set of eigenvalues of a matrix.
  - Define the eigenspace of an eigenvalue.
  - Find the corresponding space of eigenvectors associated with an eigenvalue.
  - Interpret information about the spectrum and eigenspaces to determine information about the matrix/linear transformation
  - Understand the trace and how it relates to the spectrum of a matrix
  - Understand the connection between the spectrum and eigenspaces and diagonalization.
  - Prove results relating to diagonalization and the spectrum/eigenspaces of a matrix
  - Perform steps to diagonalize a matrix.
  - Define the conjugacy operation on matrices, and the similarity relation.
  - Understand and prove the properties invariant under conjugacy.
- Inner Products
  - *To be completed...*
- Abstract Vector Spaces
  - *To be completed...*
- Applications

– *To be completed...*

Homeworks: There will be two types of homeworks in this course: WebAssign and written homework.

Written homework will, in general, be due twice a week: on Tuesday and Friday, except during exam weeks, where there will be no Tuesday homework. The week of Memorial Day and the last week of the class, the homework will be collected on Wednesday instead of Tuesday. The content on written homework will be mostly theoretical and will involve writing formal arguments. All homework problems will be labeled with a date that the problem *should* be done on. This encourages incrementally doing the homework rather than waiting to the last minute.

WebAssign will, in general, be due two days after it is assigned. Realistically, I understand that any computational algorithm that we learn in the course can be done using your computer in a fraction of the time it would take you to do by hand. I still urge you to do it by hand. The reason why is simple: I don't particularly care that you can do the arithmetic, but I do care that you can remember and carefully follow the steps of the algorithms we learn. Therefore, they will certainly make an appearance on the examinations and quizzes. If you practice these computations on WebAssign, your prospects of doing well on the exams and quizzes will greatly improve.

The following code is needed for the WebAssign:

cmu 9659 1981

Quizzes: Quizzes will the most regular form of assessment in the course. Quizzes will be (almost) everyday and will test the material covered either that day or the previous day (or both). They will only be about 10 minutes long. Because I understand that sometimes things happen that are unexpected requiring you miss class, two quizzes will be dropped. Because of this, makeups are, in general, not given.

The objective of quizzes are threefold:

- Give you feedback on my expectations and your understanding of the material thus far.
- Give myself feedback on how my expectation and teaching are working, and how much you all understand the material thus far.
- Encourage incremental studying of the material and attendance rather than cramming.

Exams: In all there are three exams. None will be purposefully cumulative, but as with any class in mathematics, material and skills are built throughout the course, and falling behind is a very bad thing.

Each exam is weighted the same.

In the event you cannot make the exam time, you must notify me with advance notice (at least 1 full week) to schedule a makeup.

The structure of the exam may not be something you are used to, but the intent is that it will test you knowledge and encourage incremental studying with the least amount of stress. Each exam will have three parts.

- The first part will be short answer. These will be relatively easy questions, which will essentially be to know definitions and do easy calculations.
- The next part will be the technique part. In advance of the test, a list of questions will be provided. From this list of questions, some questions will be chosen and put verbatim on the test.
- The last part will be the creative part. This will be questions which are easier in caliber to homework questions, and will test your understanding on how to apply these techniques to new questions you haven't seen before.

Grades: Your grade will be based on all and only the above mentioned work, namely homework, quizzes, and exams. You are more than welcomed to talk with me if you feel any of this work has been mis-graded, you feel you deserve more credit, or you think for some reason some aspect of the assessment was unfair. Here is how your grade will be calculated:

Written Homework:	15%
WebAssign	10%
Midterms	45%
Quizzes	30%

For letter grades, to guarantee a particular grade, the intent is they will be assigned according to the following scale:

A	[90,100]%
B	[80,90)%
C	[70,80)%
D	[60,70)%
R	else

Cut-offs may be adjusted (only to your benefit) at the end of the semester. At the end of the course, a report on your graded material will be sent to you.

Course Policies:

- **Attendance Policy:** Attendance is required. This will be reflected in your grade in various ways; the most direct way is through your quizzes.
- **Electronics Policy:** Electronic devices prove to be distracting to yourself and to other students during class. Therefore, you may not use them without special permission.
- **Academic Honesty:** All work handed in by you, whether in class or homework, must be the work of yourself and no one else. This will be strictly enforced. The penalty for any violation will be at least a 0 on that assignment. See the university policy on academic integrity for more information.
- **Test Resource Policy:** Outside resources are not permitted on tests. This includes any electronics, books, notes, etc.
- **Homework Resource Policy:** Because people often benefit from seeing many different sources of material, you are welcome to consult different resources not listed on the course website to study and do homework. Such consultation will not be necessary, and all assignments and tests are written to be done without any resources except the class notes and the textbook. You may not try to find and use an outside source just to find an answer to a particular problem. For example:
  - Legal Use of Outside Resources: You are stuck on a particular homework problem. Because you are stuck, you realize you may have not known that topic as well as you thought you did. The sources I have provided don't seem to be explaining it to your satisfaction, so you consult an outside source, which clears up your confusion and allows you to do the problem.
  - Illegal Use of Outside Resources: You are, as above, stuck on a particular homework problem. You try searching the question on a search engine, until you find a source which provides an answer to the question. You then paraphrase or copy the answer provided to be turned in.
- **Collaboration Policy:** You are encouraged to collaborate. Productive collaboration is a necessary skill to acquire. But, no permanent record of the discussions are allowed (we call this the *White Board Policy*). Moreover, you must cite all collaborators. The penalty for any violation will be at least a 0 on that assignment. For example:
  - Legal Collaboration: You and two friends discuss a problem, all making notes only on a white board. Once you feel like you understand the problem, the white board is erased, and you leave with no permanents notes taken from the meeting. Later, you write up the solution on your own.
  - Illegal Collaboration: You and a friend discuss a problem. You figure it out together, and then compare notes and consult each other while writing up your solutions.
- **Special Needs:** If you have documentation supporting the needs for special accommodations (extra time on tests, special seating, etc) then you must present it to me during the first week of class. I will assist with any reasonable requests.
- **Syllabus Changes:** I reserve the right to make any changes to this syllabus during the course of the semester to make the grading more fair or the course more productive.