Quiz 8

Problem. For

$$f(x) = \cos^2(x) - 2\sin(x)$$

Find the following:-

1. All local maximums and minimums
2. The absolute maximum and minimum on the interval $[0, 2\pi]$.

Solution. Taking the first derivative, we have

$$f'(x) = -2\cos(x)\sin(x) - 2\cos(x)$$

After factoring, and setting to 0, we have

$$-2\cos(x)(\sin(x) + 1) = 0$$

Notice, $\cos(x) = 0$ on $\frac{\pi}{2} + 2\pi n$ and $\frac{3\pi}{2} + 2\pi n$. Another way to write this is $\frac{\pi}{2} + \pi n$. For the other zeros, we have to solve when $\sin(x) = -1$. This happens only at $\frac{3\pi}{2} + 2\pi n$, which was already covered by the other zeros. Thus we conclude that the critical points are $\frac{\pi}{2} + \pi n$ for any $n \in \mathbb{Z}$.

Now we ask, which of these are positive, and which negative? To do this easily, we note that $\sin(x) + 1$ is never negative, and -2 is always negative. Thus the entire expression will be negative when $\cos(x)$ is positive (Quadrants I and IV) and positive when $\cos(x)$ is negative (Quadrants II and III).

$$0 \overset{-}{\text{———}} \frac{\pi}{2} \overset{+}{\text{———}} \frac{3\pi}{2} \overset{-}{\text{———}} 2\pi$$

So we see that $\frac{3\pi}{2} + 2\pi n$ is where there local maximums occurs and $\frac{\pi}{2} + 2\pi n$ is where the local minimums occur. Now these values will be calculated.

Now, we need to find the values at these two critcal points, and the endpoints.

$$f(0) = 1$$
$$f(\frac{\pi}{2}) = -2$$
$$f(\frac{3\pi}{2}) = 2$$
$$f(2\pi) = 1$$

Therefore, the location of all local mins and maxs are $(\frac{3\pi}{2} + 2\pi n, 2)$ and $(\frac{\pi}{2} + 2\pi n, -2)$. Also, we know that the absolute minimum is -2 and absolute maximum is 2 on the interval $[0, 2\pi]$ (and actually in the whole graph since it's periodic).