Quiz 2

Problem. Solve the following limits:

(a) $\lim_{h\to 0}\left(\frac{\sqrt{1+h}-1}{h}\right)$

(b) $\lim_{x\to 2}\left(\frac{x^3+x^2-4x-4}{x-2}\right)$

Solution.

(a)

$$\begin{aligned}
\lim_{h\to 0}\left(\frac{\sqrt{1+h}-1}{h}\right) &= \lim_{h\to 0}\left(\left(\frac{\sqrt{1+h}-1}{h}\right)\left(\frac{\sqrt{1+h}+1}{\sqrt{1+h}+1}\right)\right)\\
&= \lim_{h\to 0}\left(\frac{1+h-1}{h(\sqrt{1+h}+1)}\right)\\
&= \lim_{h\to 0}\left(\frac{h}{h(\sqrt{1+h}+1)}\right)\\
&= \lim_{h\to 0}\left(\frac{1}{\sqrt{1+h}+1}\right)\\
&= \frac{1}{\sqrt{1+0}+1}\\
&= \frac{1}{2}
\end{aligned}$$

(b) We divide.

$$\begin{array}{r|l}
 & x^2+3x+2 \\
x-2 & x^3 + x^2 - 4x - 4 \\
 & -x^3 + 2x^2 \\
 & 3x^2 - 4x \\
 & -3x^2 + 6x \\
 & 2x - 4 \\
 & -2x + 4 \\
 & 0
\end{array}$$

Then we can evaluate the limit fairly easily.

$$\begin{aligned}
\lim_{x\to 2}\left(\frac{x^3+x^2-4x-4}{x-2}\right) &= \lim_{x\to 2}\left(\frac{(x-2)(x^2+3x+2)}{x-2}\right)\\
&= \lim_{x\to 2}\left(x^2+3x+2\right)\\
&= 12
\end{aligned}$$

Alternatively to division, in this problem it is actually factorable.

$$\begin{aligned}
x^3+x^2-4x-4 &= x^2(x+1)-4(x+1)\\
&= (x+1)(x^2-4)\\
&= (x+2)(x-2)(x+1)
\end{aligned}$$