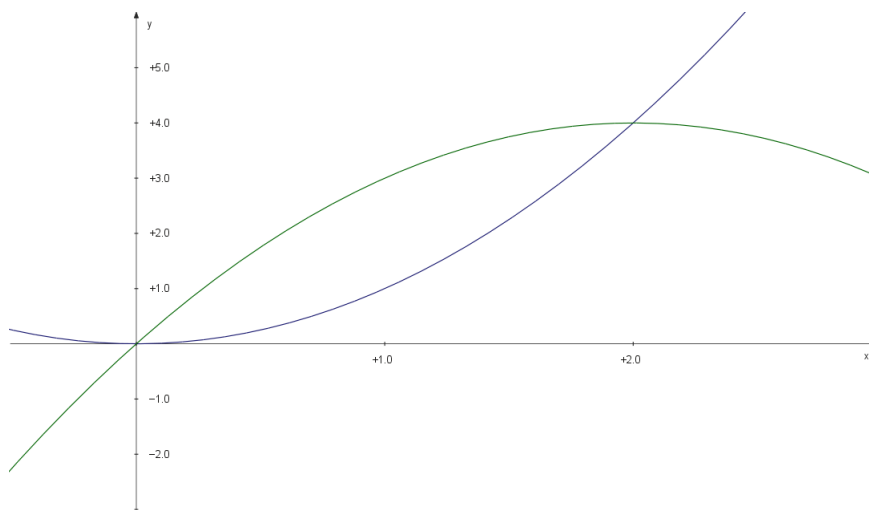


Quiz 16

Problem. Find the area bounded by the curves

$$y = x^2$$
$$y = 4x - x^2$$

Solution. Here is the graph of this boundary:



We need to know where they intersect. So, we set them equal, and solve.

$$x^2 = 4x - x^2$$
$$\implies 2x^2 - 4x = 0$$
$$\implies 2x(x - 2) = 0$$
$$\implies x = 0 \text{ or } x = 2$$

Thus, the points of intersection are $(0, 0)$ and $(2, 4)$. We make slices perpendicular to the x -axis (for no other reason other than to avoid having to find the inverse of $y = 4x - x^2$) and note that $y = 4x - x^2$ is the top function. Thus, to find the area we integrate

$$\int_0^2 ((4x - x^2) - (x^2)) dx = \int_0^2 (4x - 2x^2) dx$$
$$= [2x^2 - \frac{2}{3}x^3]_0^2$$
$$= 2(2)^2 - \frac{2}{3}(8)$$
$$= 8(1 - \frac{2}{3})$$
$$= \frac{8}{3}$$