21-120 Differential and Integral Calculus Summer Session I 2010

Midterm Exam Review

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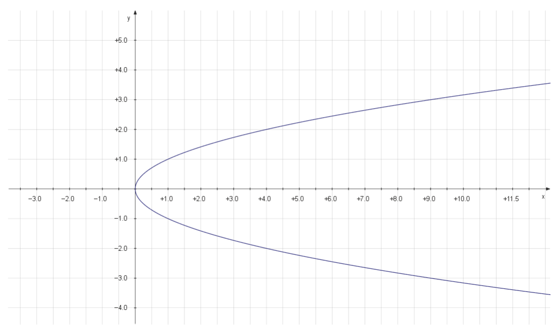
This list is not comphrensive. You are expected to be able to answer any question using the tools that I have taught in class. Here is an outline of the major concepts taught in class, along with at an example to demonstrate the concept (for examples of §2.7 and §4.5, see the book).

The test will be 20 questions, and there will be two parts. The first part will be 10 free response-type questions each worth 10 points. Partial credit will be given on this section for work shown. The second part is 10 multiple choice and short answer-type questions, each worth 5 points. Partial credit will not be given on these problems.

1. §1.1-2: Identifying whether a given rule or graph is or is not a function.
   *Example:* Is $f(x) = \sin(x)$ a function? Is the following graph a function?



2. §1.3: Being able to decide whether a given limit exists
   *Example:* Does $\displaystyle\lim_{x\to\infty} \frac{x}{x+3}$ exist? How about $\displaystyle\lim_{x\to -3} \frac{1}{x+3}$ ?

3. §1.3-4,1.6,3.7: Calculating limits

   (a) Involving infinity
   *Example:* $\displaystyle\lim_{x\to\infty} e^{-1/x^2}$

   (b) Involving removable discontinutites
   *Example:* $\displaystyle\lim_{x\to -2} \frac{x^2-4}{x+2}$

   (c) Using Squeeze Theorem
   *Example:* $\displaystyle\lim_{x\to 0} \frac{\sin(1/x)}{x}$

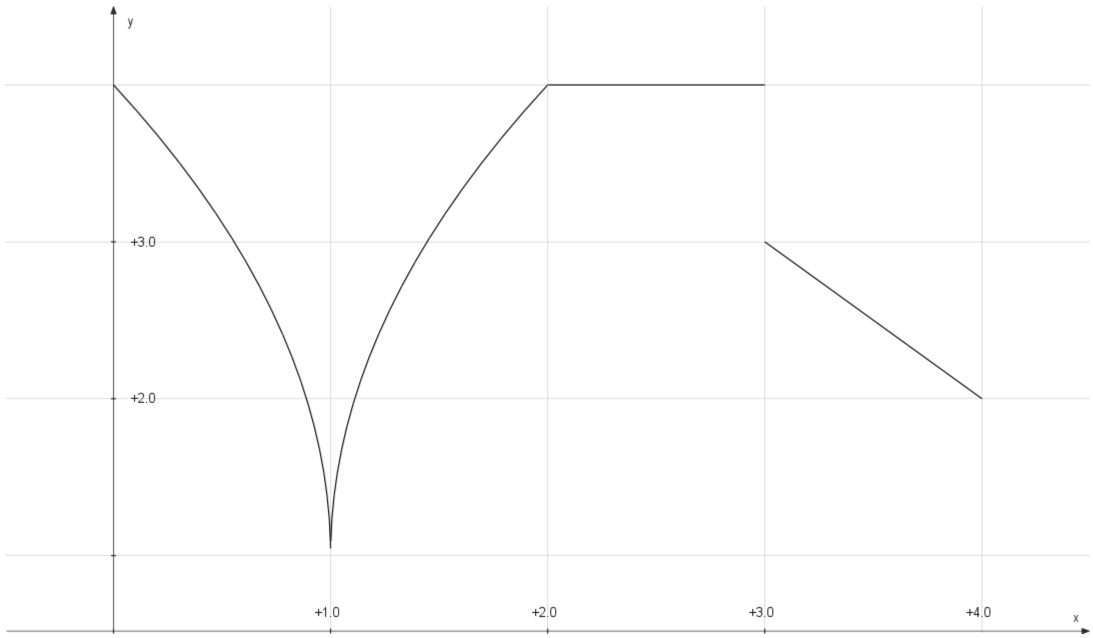
   (d) Of things like $\displaystyle\frac{\sin(x)}{x}$
   *Example:* $\displaystyle\lim_{x\to 0} \frac{\sin(3x^2)}{x^2}$

   (e) Using L'Hôpital's Rule
   *Example:* $\displaystyle\lim_{x\to 0} \frac{1-\cos(x)}{\sin(x)}$

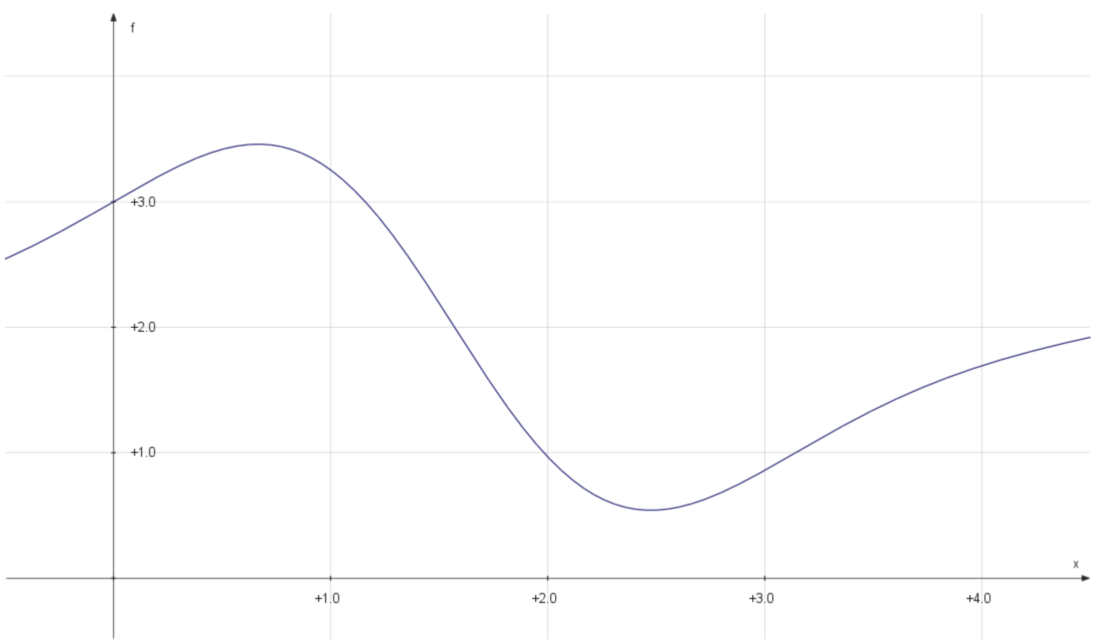
4. §2.1-5: Knowing how to take the derivative of any function involving products, sums, difference, and quotients or polynomials, logs, exponentials, trig, and inverse trig functions.
   *Example:* Find $f'(x)$ if $f(x) = \dfrac{e^{\sin(x)}}{\cos(\ln(x))} \cdot \sin^{-1}(\ln(\cos(x^2)))$

5. §2.6: Taking the derivative of an implicit function.
   *Example:* Find $y'$ if $y = \cos(y)\sin(x)^2 - y^2$

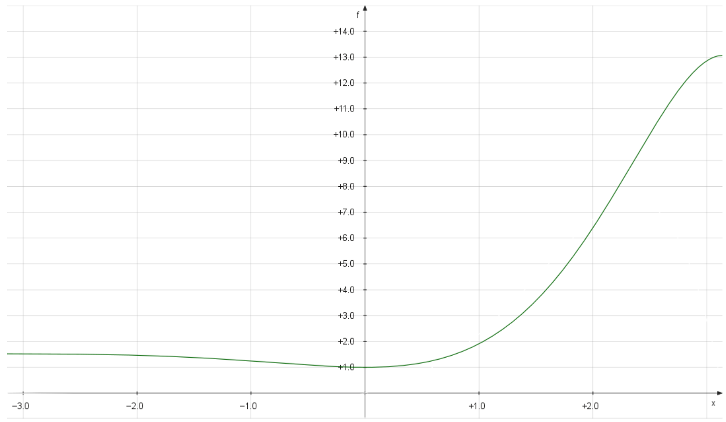
6. §1.5,2.1-2: Identifying whether a function is continuous or differentiable at any point or interval
   *Example:* Is the following graph differentiable on $(0, 1)$? How about $(.5, 1.5)$? How about $(1.5, 2.5)$? How about $(2.5, 3.5)$? How about $(2, 3)$? How about continuous on all those intervals?




7. §2.1-2: Being able to answer questions about $f'$ given a graph of $f$ and visa versa.
   *Example:* In the following graph, if the function is $f$, what is the sign of $f'$ at $x = 0, 1, 2, 3, 4$? How about $f''$? If the graph is instead $f'(x)$ what can you say about whether $f$ is increasing at $x = 3$?



8. §2.1-2: Being able to identify the graph of the derivative given the graph of the function, and visa versa.
*Example:* The following is the graph of $f$. Which is it's derivative $f'$?



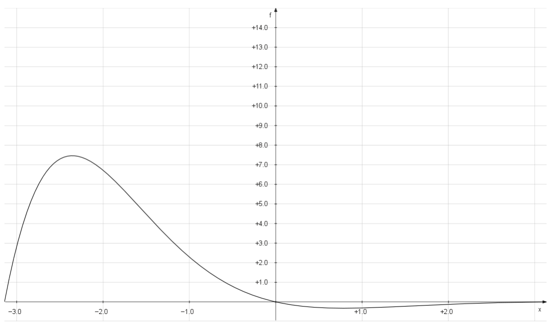



Figure 1: A

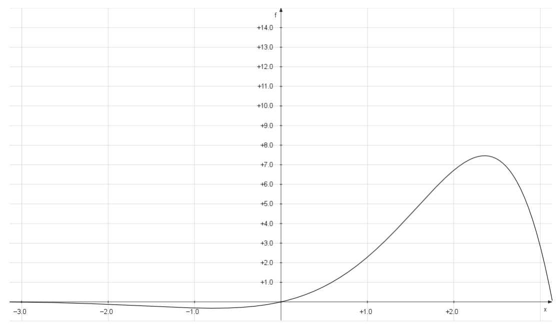


Figure 2: B

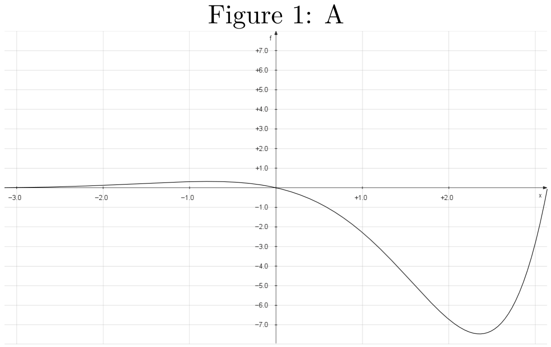


Figure 3: C

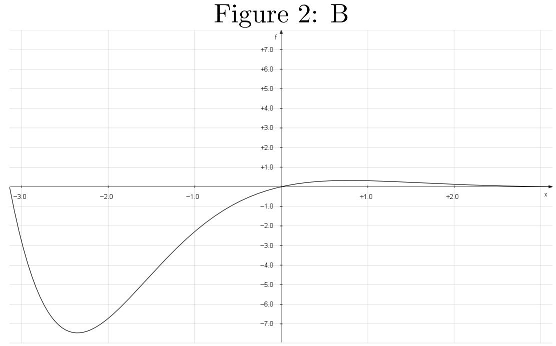


Figure 4: D

9. §2.7: Related Rates Problems (including knowing all the formulas).
*Example:* §2.7 problems from book.

10. §2.8: Approximating a function by a linear function.
*Example:* Approximate $f(x) = x^2 \sin(2x)$ near $x = 0$ by a linear function. Give an approximation for $\sqrt[3]{8.02}$.

11. §3.4: Exponential Growth/Decay Problems
    *Example:* The rate of growth of a colony of bacteria is proportional to the population. If a population of bacteria has 200 members after 1 hour, and 400 members have 2 hours, how many members will it have after 6 hours?

12. §4.1,4.3: Finding local and absolute maximum and minimums of a function.
    *Example:* Given $f(x) = \sin(x) \cdot e^x$, final the location of all local minimums and maximums. Find the absolute minimum and maximum on the interval $[-\pi, \pi]$.

13. §4.3-4: Find the intervals a function is increasing, decreasing, concave up, and concave down, and finding all points of inflection. Also, putting all this information together to be able to sketch a graph.
    *Example:* Given $f(x) = \sin(x) \cdot e^x$ sketch the graph on the interval $[-\pi, \pi]$

14. Know what the Mean Value Theorem and Rolle's Theorem states. (the assumptions and the conclusions).

15. §4.5: Optimization Problems.
    *Example:* §4.5 problems from book.