Spring 2024 **Functional Analysis 21-640** **CMU**

Lecture: MWF 10:00 am – 10:50 am, Wean Hall 8220

Lecturer: Tomasz Tkocz, Wean Hall 8117, `ttkocz@math.cmu.edu`

TA: Mihalis Sarantis, `msaranti@andrew.cmu.edu`

Office Hours: M 11 am – 12:30 pm or by email appointment

Course website: `Canvas` and/or `http://math.cmu.edu/~ttkocz`

Course description: This course is a rigorous introduction to functional analysis, starting from the notion of Banach space, entering a marvelous world, full of wonderful insights, where continuous linear bijections' inverses are automatically continuous as well.

Prerequisites: Linear algebra, real analysis, and topology, also useful would be some basics of complex analysis.

Literature:

- Eidelman, Y., Milman, V., Tsolomitis, A., Functional analysis: an introduction. *AMS Graduate Studies in Mathematics*, 2004.
- Stein, E., Shakarchi, R., Functional Analysis : Introduction to Further Topics in Analysis *Princeton University Press*, 2011.

Course content: Linear spaces, normed spaces, first examples and constructions of Banach spaces; Hilbert spaces; L_p spaces, Riesz' interpolation; The Hahn-Banach lemma and separation of convex sets; Topological "miracles": the uniform boundedness principle, the open mapping and the closed graph theorems; Weak topologies; Basics of operator theory: Fredholm theory, self-adjoint operators, spectral decomposition and functions of operators;

Learning objectives: Students should

- gain understanding of fundamental concepts concerning complete linear spaces with norms and inner products, and maps between such
- advance their insight into the interplay between geometric and analytic ideas of infinite dimensional normed vector spaces
- develop an improved ability and use the methods and results of functional analysis, with applications in other areas, particularly classical analysis (Fourier series), geometry, differential equations

Course format: This is an in person class. You are expected to fully participate in class, viz. please ask and answer questions, initiate or participate in discussions.

Homework: There will be about 12 homework assignments during the semester.

Late submissions will not be accepted, but the lowest homework score will not count towards the final grade. Plagiarism is not tolerated. Collaboration on homework is allowed, but has to be acknowledged in writing and the solutions must be written on your own, at least one tea break after the collaboration ended.

The assignments will be administered via Gradescope. Only high quality pdf-scans of hand-written solutions will be accepted (consider apps like Dropbox, or Notes on iOS to produce them), or use LaTeX.

Exams: There will be 3-4 in-class tests throughout the semester (based on the practice problems and the lecture material). *No* final exam, *but* suggested grades will be out before the end of the semester and you can request an oral final examination to improve your grade. Plagiarism and cheating are not tolerated.

Grades: The midterm grade will be based solely on homework. The final grade will be based on homework and tests, computed as a weighted average:

30% **Homework** + 70% **Tests**

Functional analysis of infinite dimensional spaces is never fully convincing;
you don't get a feeling of having done an honest day's work.

–Gian-Carlo Rota [on combinatorics]

Combinatorics is discrete functional analysis in my world view,
while functional analysis is applied combinatorics.

–Bill Johnson

War is the continuation of policy with other means.

–Carl von Clausewitz

Analysis is continuation of geometry with other means.

–unknown