Probability 21-325 **Homework 1 (due 23rd Jan)** **TT**

1. How many different sequences do we obtain by permuting the letters of the following words: a) DERMATOGLYPHICS b) INTESTINES c) CHINCHERINCHEE ? Explain.

2. Consider the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 15$. How many nonnegative integer solutions does it have? How about positive integer solutions? How many nondecreasing functions $f\colon \{1, 2, \ldots, 15\} \to \{1, 2, 3, 4, 5\}$ are there? *Hint: oranges and boxes.*

3. You are dealt a poker hand of five cards from a regular deck of 52. What is the chance that you get two pairs (but not four of a kind or a full house)?

4. A certain planet has n days in one year. What is the probability that among k people on that planet there are (at least) two who share their birthday?

5. Prove that for any events $A_1, A_2, \ldots$ we have

$$\mathbb{P}\left(\bigcup_{i=1}^{\infty} A_i\right) \leq \sum_{i=1}^{\infty} \mathbb{P}\left(A_i\right).$$

6. Prove that for any events $A_1, A_2, A_3, \ldots, A_n$ we have the *inclusion-exclusion* formula

$$\mathbb{P}\left(A_1 \cup A_2 \cup \ldots \cup A_n\right) = \sum_{1 \leq i \leq n} \mathbb{P}\left(A_i\right) - \sum_{1 \leq i < j \leq n} \mathbb{P}\left(A_i \cap A_j\right) + \sum_{1 \leq i < j < k \leq n} \mathbb{P}\left(A_i \cap A_j \cap A_k\right) - \ldots + (-1)^{n-1} \mathbb{P}\left(A_1 \cap A_2 \cap \ldots \cap A_n\right).$$

7. Suppose that events A and B satisfy $\mathbb{P}(A \cup B) = 1/2$, $\mathbb{P}(A \cap B) = 1/4$ and $\mathbb{P}(A \setminus B) = \mathbb{P}(B \setminus A)$. Find $\mathbb{P}(A)$.

8. Suppose that events A, B and C satisfy $\mathbb{P}(A \cap B \cap C) = 0$ and each of them has probability not smaller than $2/3$. Find $\mathbb{P}(A)$.

9. There are n pairs of shoes in a closet. Pick at random k shoes ($k < n$). What is the probability that a) at least one pair of shoes has been picked b) exactly one pair of shoes has been picked?

10. Let $A_1, A_2, \ldots, A_n$ be elements of a σ-field $\mathcal{F}$. Show that for every $k \in \{1, 2, \ldots, n\}$ the set of all elements which belong to exactly k of the A_i is also an element of $\mathcal{F}$.