Probability 21-325 **Homework 8 (due 4th April)** **TT**

1. Show that if $X_n \xrightarrow[n\to\infty]{\mathbb{P}} X$ and $X_n \xrightarrow[n\to\infty]{\mathbb{P}} Y$, then $\mathbb{P}(X = Y) = 1$ (in other words, the limit in probability is unique).

2. Let X be an integrable random variable and define

$$X_n = \begin{cases} -n, & X < -n \\ X, & |X| \leq n \\ n, & X > n. \end{cases}$$

Does the sequence X_n converge a.s., in L_1, in probability?

3. Let $X_1, X_2, \ldots$ be i.i.d. integrable random variables. Prove that $\frac{1}{n}\max_{k\leq n}|X_k|$ converges to 0 in probability.

4. Show that if $X_n \xrightarrow[n\to\infty]{\mathbb{P}} X$ and $Y_n \xrightarrow[n\to\infty]{\mathbb{P}} Y$, then $X_n + Y_n \xrightarrow[n\to\infty]{\mathbb{P}} X + Y$.

5. Show that if $X_n \xrightarrow[n\to\infty]{\mathbb{P}} X$ and $Y_n \xrightarrow[n\to\infty]{\mathbb{P}} Y$, then $X_nY_n \xrightarrow[n\to\infty]{\mathbb{P}} XY$.

6. Prove that a sequence of random variables X_n converges a.s. if and only if for every $\varepsilon > 0$, $\lim_{N\to\infty}\mathbb{P}\left(\bigcap_{n,m\geq N}|X_n - X_m| < \varepsilon\right) = 1$ (the Cauchy condition).

7. Does a sequence of independent random signs $\varepsilon_1, \varepsilon_2, \ldots$ converge a.s.?

8. Let $X_1, X_2, \ldots$ be independent random variables, $X_n \sim \text{Poiss}(1/n)$. Does the sequence X_n converge a.s., in L_2, in probability?