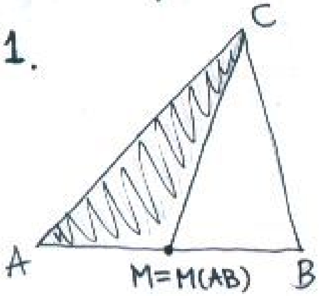
Kółko w gimnazjum Staszica

„Równe pola"

6 IV 2010

1.



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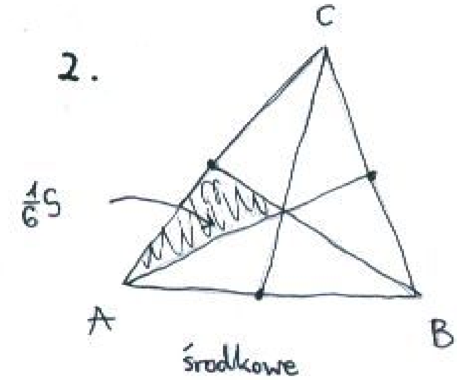

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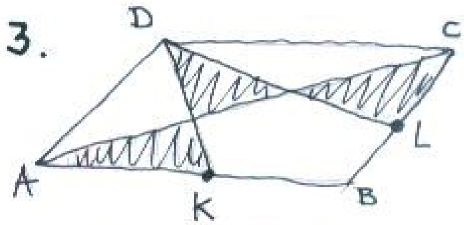
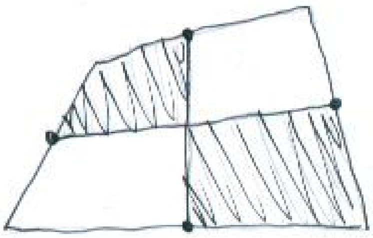


K, L - środki $\Rightarrow$ ▨ $= \frac{1}{3}[ABCD]$

4.



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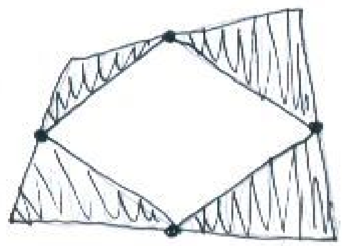

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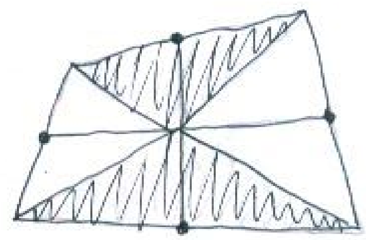

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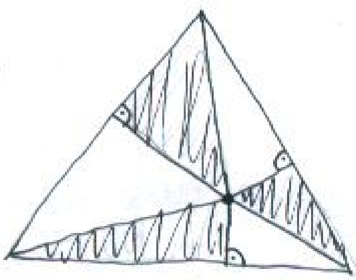

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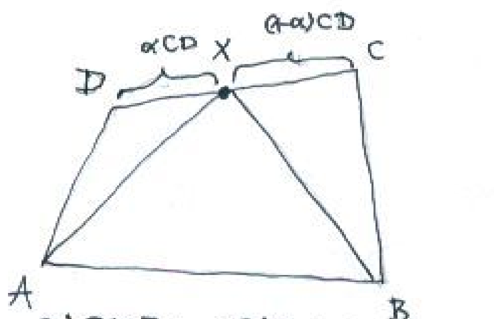
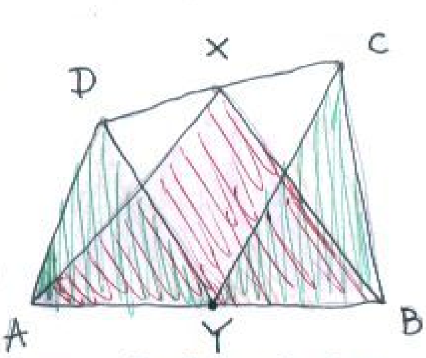


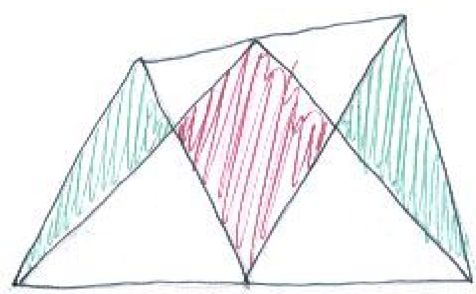
$[ABX] = \alpha[ABC] + (1-\alpha)[ABD]$

9.



$[ABX] = [AYD] + [YBC]$

10.



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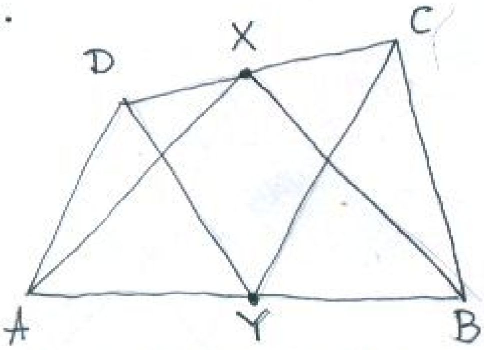
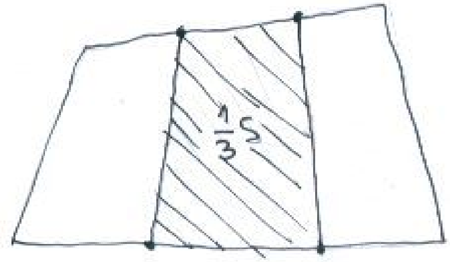


$[ABX] + [CDY] = [ABCD]$

12.



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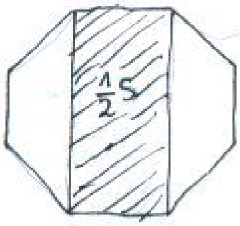

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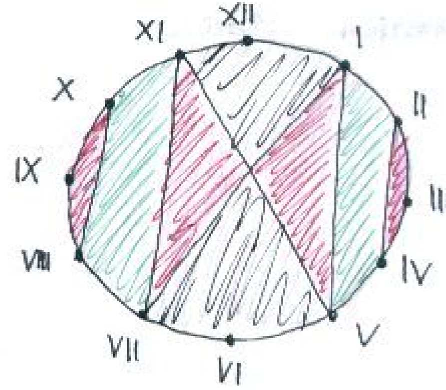
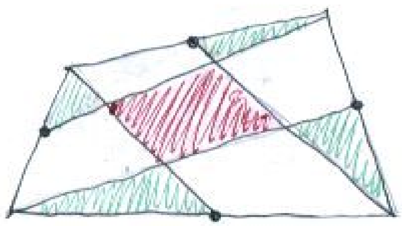


Który kolor zajmuje najwięcej miejsca?

15.



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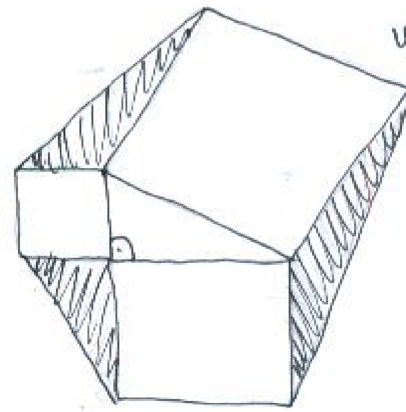
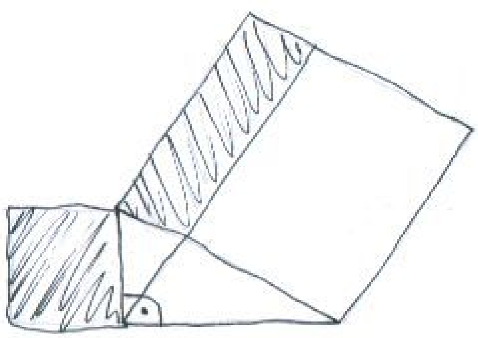


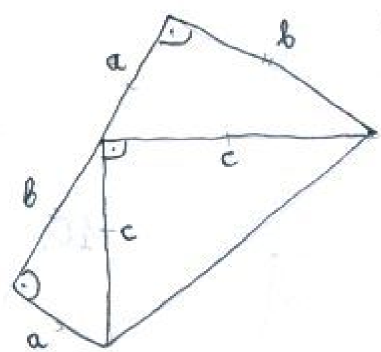
Ud. że Pola szarych Δ są jednakowe

17.



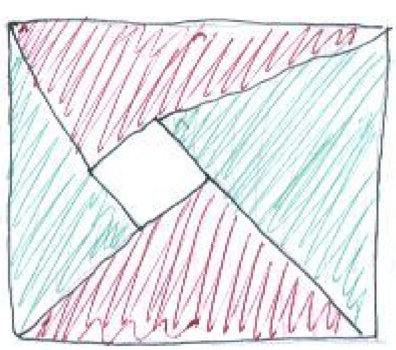
+ wn. tw. Pitagorasa

18.

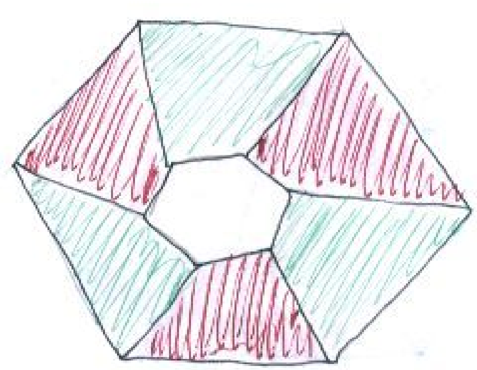


· Oblicz pole trapezu na 2 sposoby dowodząc, że $a^2+b^2=c^2$.

19.



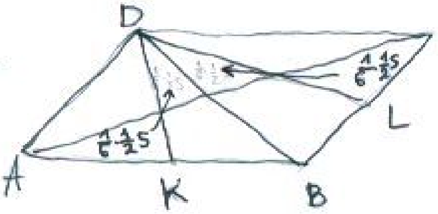
20.



Rozwiązania

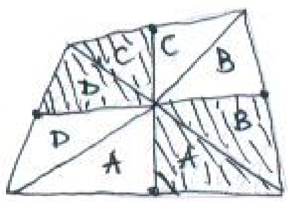
1, 2. ⊙⊙

3.



Prowadzimy przekątną BD i korzystamy z 2.

4.



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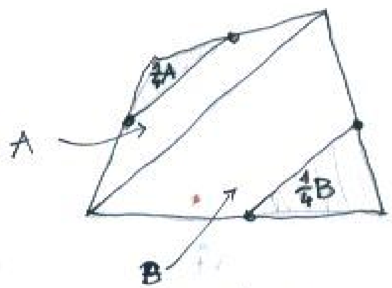
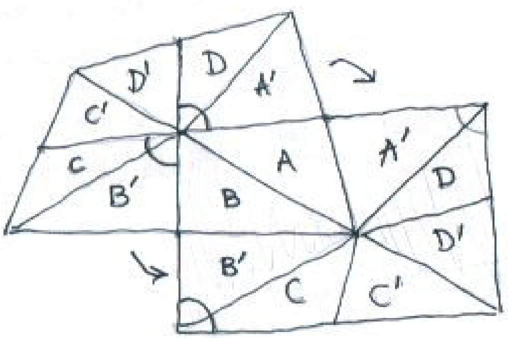
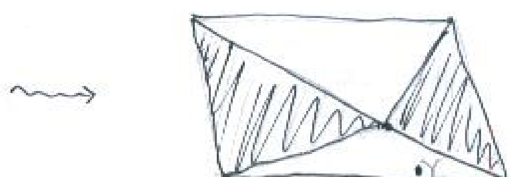


$A+B=S$

Czyli zadanie sprowadziliśmy do oczywistej sytuacji w równoległoboku

6.





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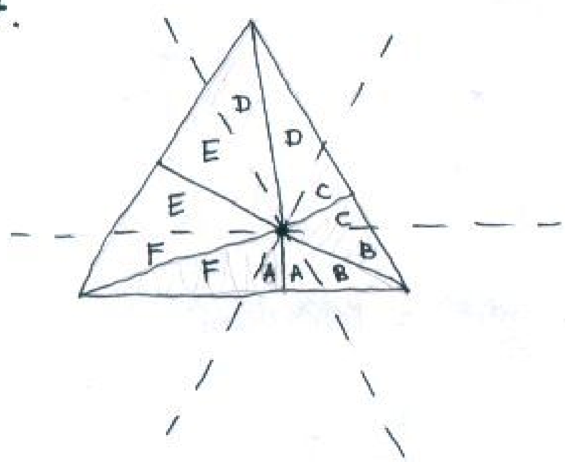

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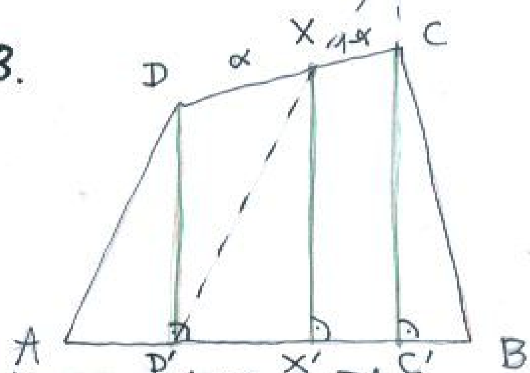


Liczymy wysokości z Talesa

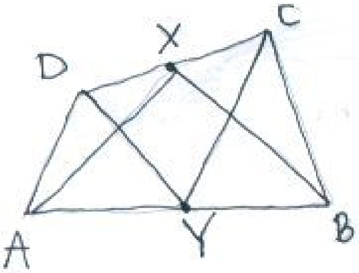
$$XX' = YC' \cdot \frac{D'X'}{D'C'} = (YC + CC')\alpha$$
$$= \alpha YC + \alpha CC' = \alpha \cdot XC \cdot \frac{DD'}{XD} + \alpha CC' = \alpha \cdot CC' + (1-\alpha)$$

stąd łatwo teza.

9. ⊙⊙ wn z 10. (przy pomocy 1.)

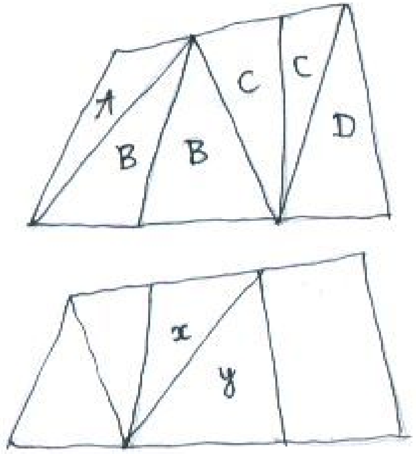
10. ⊙⊙ wn z 9.

11.



$[ABX] = [ABCD] - ([ADX] + [XCB]) \underset{9.}{=} [ABCD] - [CDY]$.

12.

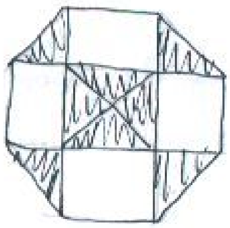


$$x = \frac{A+C}{2} \quad \Rightarrow \quad \underset{B+C}{\overset{\parallel}{x+y}} = \frac{A+B+C+D}{2} \quad \Rightarrow \quad B+C = A+D$$
$$y = \frac{B+D}{2}$$

$$\Downarrow$$

$$B+C = \frac{1}{3}(A+2B+2C+D) = \frac{1}{3} S.$$

13.



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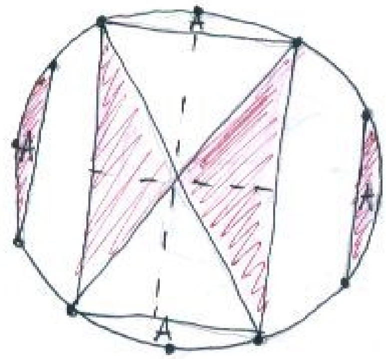
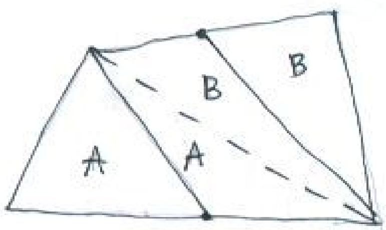
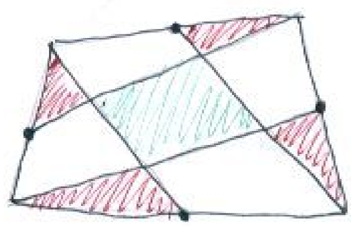


Odp. Wszystkie kolory zajmują tyle samo miejsca.
To, że szary jest ☺☺
Widać, że fiolet = szary

15.



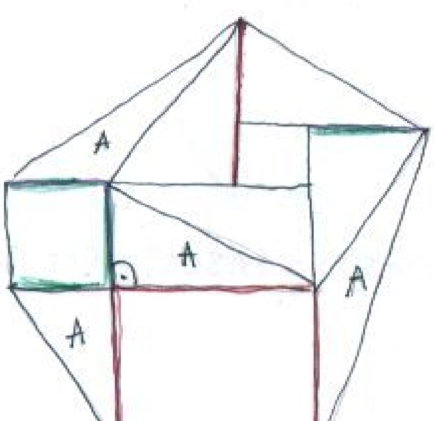


czyli pole paska to $\frac{1}{2}S$

Mamy zaś

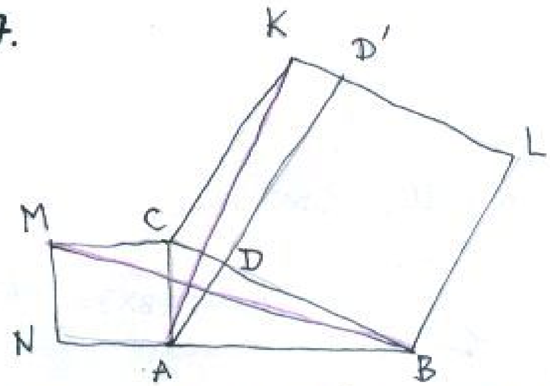
$$|\text{czerwone}| = S - |\text{Pasek 1} \cup \text{Pasek 2}| = S - \big(\underbrace{|\text{Pasek 1}| + |\text{Pasek 2}|}_{S} - |\text{zieleń}|\big) = \text{zieleń}$$

16.



Każdy z szarych Δ ma pole równe A.

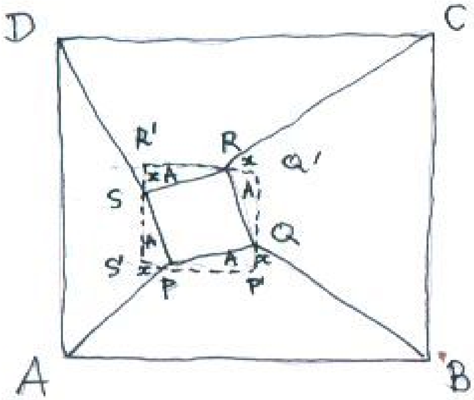
17.



$[CDD'K] = [AKK] = [CMB] = [ACMN]$.

18. łatwe

19.



Dla nieobróconego kwadratu P'Q'R'S' jest jasne, iż

$$[ABP'P] + [CDR'R] \overset{(*)}{=} [BCQ'Q] + [DAS'S]$$

$$\| \qquad\qquad\qquad\qquad\qquad \|$$

$$[ABQP] + [BQP'] - A + [CDSR] + [DSR'] - A$$

$$[BCRQ] + [CRQ'] - A + [DAPS] + [APS'] - A$$

ale $[BQP'] + [DSR'] = \frac{1}{2}x \cdot \left(dist(B, P'Q') + dist(D, S'R')\right)$

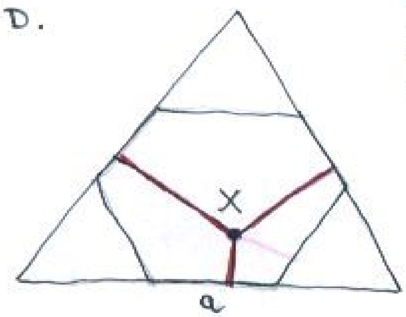
$= \frac{1}{2}x \cdot \left(dist(C, R'Q') + dist(A, S'P')\right)$

$= [CRQ'] + [APS']$

więc w (*) się skraca i wychodzi teza.

20. LM. Suma odległości dowolnego punktu wewnętrznego 6-kąta foremnego od co drugiego boku jest stała.

D-D.

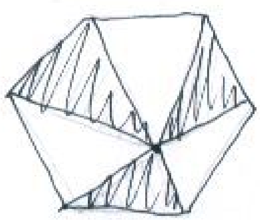


Wrysujemy 6kąt w trójkąt równoboczny jak na rys. i widzimy, że owa suma odległości jest sumą odległości punktu X od boków tego trójkąta i jak wiadomo wynosi

$$\frac{S}{\frac{1}{2}3a} = \frac{\frac{(3a)^2\sqrt{3}}{4}}{\frac{1}{2}3a} = \frac{3a\sqrt{3}}{2}.$$

Przy okazji mamy

WN.



Dalej postępujemy już jak w 19.