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On the Quasi-convexity of a functional in Nonlinear Elasticity

Abstract: Let $\Omega \subset \mathbb{R}^3$ denote the reference configuration of a nonlinear elastic material, and consider deformations$\boldsymbol{u} : \Omega \to \mathbb{R}^3$. We determine a sufficient condition for the stored energy density $W(\mathbf{F}) = |\mathbf{F}|^p + h(\det \mathbf{F})$, where h is a smooth and convex function, and $2 < p < 3$, to be $W^{1,p}$-quasiconvex at $\lambda \mathbf{I}$ on the restricted class of deformations $\boldsymbol{u}$ that satisfy condition (INV), $\det \nabla \boldsymbol{u} > 0$ a.e., and which open a single hole anywhere in the material. The condition takes the form $\lambda^{3-p} h'(\lambda^3) \leq \Upsilon_p$, where $\Upsilon_p > 0$ is explicitly determined, and is in a certain sense optimal for the class of energy densities under consideration. The proof makes use of isoperimetric inequalities and of estimates on radial p-harmonic functions.