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Optimal Lifting for BV Functions into S^1

Abstract: Let $\Omega \subset \mathbb{R}^N$ be an open set and $u : \Omega \to S^1$ a measurable function. A lifting of u is a measurable function $\varphi : \Omega \to$
$mathbbR$ such that $u(x) = e^{i\varphi(x)}$ for a.e. $x \in \Omega$. If u has some regularity one may ask whether or not φ can be chosen with some regularity as well. Motivated by the study of the Ginzburg-Landau equation and the degree theory, there has been recently much research on this topic for certain spaces of functions with values into the circle S^1 (such as Sobolev spaces, BMO, VMO, BV).

The aim of this talk is to construct an optimal lifting for $u \in BV(\Omega, S^1)$ and to determine its total variation, i.e.,

$$E(u) = \min\left\{ \int_\Omega |D\varphi| \,:\, \varphi \in BV(\Omega, \mathbb{R}),\ e^{i\varphi} = u \text{ a.e. in } \Omega \right\}. \qquad (1)$$

The formula of $E(u)$ will involve the notion of minimal connection between surface singularities of dimension $N-2$ of u. We also discuss the relation between optimal BV lifting and minimizers of the $\Gamma-$limit of the functionals $\{F_\varepsilon\}_{\varepsilon \downarrow 0}$ defined as

$$F_\varepsilon(\varphi) = \varepsilon \int_\Omega |\nabla\varphi|^2 + \frac{1}{\varepsilon}\int_\Omega |u - e^{i\varphi}|^p, \quad \forall \varphi \in H^1(\Omega, \mathbb{R}) \qquad (2)$$

where $p \in (1, +\infty)$.