which gives the possibility of expressing the term in the asymptotics (1) of the function $E_\varepsilon(k_{11})$.

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ASYMPTOTIC STABILITY OF LINEAR STOCHASTIC DIFFERENTIAL EQUATIONS OF NEUTRAL TYPE

L. E. Shaikhet

We consider a scalar linear stochastic differential equation of neutral type

$$\dot{x}(t)+ax(t)+bx(t-h)+c\dot{x}(t-h)+\sigma x(t-\tau)\dot{w}(t)=0 \tag{1}$$

with initial condition $x(s) = \varphi_0(s)$, $s \le 0$. Here $w(t)$ is a standard Wiener process, a, b, σ are arbitrary constants, $|c| < 1$, $\varphi_0 \in H_0$, H_0 is the set of random functions $\varphi_0(s)$, $s \in [-h_0, 0]$, $h_0 = \max[h, \tau]$, right continuous and having left limits, with the norm $\|\varphi_0\|^2 = \sup_s \mathbf{M}|\varphi_0(s)|^2$.

With the aid of the method of Lyapunov functionals one has obtained the domains of variation of the coefficients a and b of Eq. (1) (for various c and σ) for which the trivial solution of this equation is mean square asymptotically stable.

We mention that for $c = 0$ or $\sigma = 0$ Eq. (1) has been for long the classical example for the illustration of domains of stability of this kind (see, for example, [1-4]). The stability conditions obtained here form a natural generalization of the known results on stochastic differential equations of neutral type.

We consider the functional

$$\begin{aligned} V(t, x_t) = {} & (x(t)+cx(t-h))^2+\nu\Big(x(t)+cx(t-h)- \\ & -b\int_{t-h}^{t} x(s)\,ds\Big)^2 + \nu|b(a+b)|\int_{t-h}^{t}(s+h-t)x^2(s)\,ds + \\ & + A_0\int_{t-h}^{t} x^2(s)\,ds + (\nu+1)\sigma^2\int_{t-\tau}^{t} x^2(s)\,ds, \end{aligned} \tag{2}$$

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where $A_0 = |ca + b + \nu c(a + b)| + 2|bc|\rho(bc)$, $\nu \ge 0$, $\rho(s) = 0$ for $s \ge 0$ and $\rho(s) = 1$ for $s < 0$, x_t is the trajectory of the process $x(s)$ for $s \le t$.

From [3] it follows that for the mean square asymptotic stability of the trivial solution of Eq. (1) it is sufficient that for some $k > 0$ and any $0 < t_1 < t_2$ the functional (2) should satisfy the condition

$$MV(t_2, x_{t_2}) - MV(t_1, x_{t_1}) \leqslant -k \int_{t_1}^{t_2} Mx^2(s)\,ds. \qquad (3)$$

We obtain conditions on the parameters of Eq. (1), under which the functional (2) satisfies the inequality (3) and which, consequently, are sufficient for the mean square asymptotic stability of the trivial solution of Eq. (1).

Making use of Itô's stochastic differentiation [5], we obtain that

$$\begin{aligned} dV(t, x_t) = \Big[& x^2(t)(-2a - 2\nu(a+b) + (\nu+1)\sigma^2 + A_0 + \nu h|b(a+b)|) - \\ & -2x(t)x(t-h)(ca + b + \nu c(a+b)) + x^2(t-h)(-2bc - A_0) + \\ & + 2\nu b(a+b)\int_{t-h}^{t} x(t)x(s)\,ds - \nu|b(a+b)|\int_{t-h}^{t} x^2(s)\,ds\Big]dt + \\ & + 2\sigma x(t-\tau)\Big[(\nu+1)(x(t) + cx(t-h)) - \nu b\int_{t-h}^{t} x(s)\,ds\Big]dw(t). \end{aligned}$$

From here for $p = \sigma^2/2$ it follows that

$$\begin{aligned} & MV(t_2, x_{t_2}) - MV(t_1, x_{t_1}) \leqslant \\ & \leqslant 2\int_{t_1}^{t_2} [Mx^2(s)(-a - \nu(a+b) + (\nu+1)p + \nu h|b(a+b)| + \\ & + |ca + b + \nu c(a+b)| + |bc|\rho(bc)) - Mx^2(s-h)(bc + |bc|\rho(bc))]\,ds. \end{aligned}$$

Since $bc + |bc|\rho(bc) \ge 0$, for the mean square asymptotic stability it is sufficient that for some $\nu \ge 0$ we have the inequality

$$a + \nu(a+b) > (\nu+1)p + \nu h|b(a+b)| + |ca + b + \nu c(a+b)| + |bc|\rho(bc),$$

which can be rewritten in the form

$$\begin{aligned} & a + b > p + \\ & + \inf_{\nu \ge 0} \frac{\nu h|b(a+b)| + |ca + b + \nu c(a+b)| + |bc|\rho(bc) + b}{\nu + 1}. \end{aligned} \qquad (4)$$

Since $|ca + b + \nu c(a + b)| \le |b|(1 - c) + (\nu + 1)|c(a + b)|$, strengthening somewhat the inequality (4), we obtain

$$\begin{aligned} & a + b > p + |c(a+b)| + \\ & + \inf_{\nu \ge 0} \frac{\nu h|b(a+b)| + |+ bc|\rho(bc) + b + |b|(1-c)}{\nu + 1}. \end{aligned} \qquad (5)$$

It is easy to see that the right-hand side of the inequality (5) is nonnegative. Consequently, $a + b > 0$ and the inequality (5) can be rewritten in the form

$$\begin{aligned} & (a+b)(1 - |c|) > p + \\ & + \inf_{\nu \ge 0} \frac{\nu h|b|(a+b) + |bc|\rho(bc) + b + |b|(1-c)}{\nu + 1}. \end{aligned} \qquad (6)$$

We investigate the inequality (6). Assume first that $bc \ge 0$. Then $\rho(bc) = 0$ and inequality (6) assumes the form

$$(a+b)(1 - |c|) > p + \inf_{\nu \ge 0} \frac{\nu h|b|(a+b) + b + |b|(1-c)}{\nu + 1}. \qquad (7)$$

Let $b \le 0$, $c \le 0$. From (7) it follows that

$$(a+b)(1+c)>p+\inf_{\nu\geqslant 0}\frac{\nu h(a+b)+|c|}{\nu+1}|b|.$$

If $h(a+b)\leq|c|$, then the infimum is attained for $\nu=\infty$ and the stability condition is described by the inequality $(a+b)(1+c+bh)>p$. If $h(a+b)>|c|$, then the infimum is attained for $\nu=0$. Then $a(1+c)>p-b$.

Let $b\geq 0$, $c\geq 0$. From (7) it follows that

$$(a+b)(1-c)>p+\inf_{\nu\geqslant 0}\frac{\nu h(a+b)+2-c}{\nu+1}b.$$

As before, we obtain $(a+b)(1-c-bh)>p$ for $h(a+b)\leq 2-c$ and $a(1-c)>p+b$ for $h(a+b)>2-c$.

Assume now that $bc<0$. Then $\rho(bc)=1$ and inequality (6) assumes the form

$$(a+b)(1-|c|)>p+\inf_{\nu\geqslant 0}\frac{\nu h|b|(a+b)+(b+|b|)(1-c)}{\nu+1}. \quad (8)$$

In the case $b<0$, $c>0$ from (8) we obtain $(a+b)(1-c)>p$. Let $b>0$, $c<0$. From (8) it follows that

$$(a+b)(1+c)>p+\inf_{\nu\geqslant 0}\frac{\nu h(a+b)+2(1-c)}{\nu+1}b.$$

Consequently, $(a+b)(1+c-bh)>p$ for $h(a+b)\leq 2(1-c)$ and $a(1+c)>p+b(1-3c)$ for $h(a+b)>2(1-c)$.

Thus, sufficient conditions of the mean square asymptotic stability of the trivial solution of Eq. (1) are determined by the following inequalities:

1) $0\leqslant c<1$;
a) $b\leqslant 0$, $\quad a>p/(1-c)-b$;
b) $0<b<(1-c)/h-p/(2-c)$, $\quad a>p/(1-c-bh)-b$;
c) $b\geqslant(1-c)/h-p/(2-c)$, $\quad a>(p+b)/(1-c)$;

2) $0>c>-1$;
a) $0\geqslant b>-(1+c)/h-p/c$, $\quad a>p/(1+c+bh)-b$;
б) $b\leqslant-(1+c)/h-p/c$, $\quad a>(p-b)/(1+c)$;
c) $0<b<(1+c)/h-p/2/(1-c)$, $\quad a>p/(1+c-bh)-b$;
d) $b\geqslant(1+c)/h-p/2/(1-c)$, $\quad a>(p+b(1-3c))/(1+c)$.

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