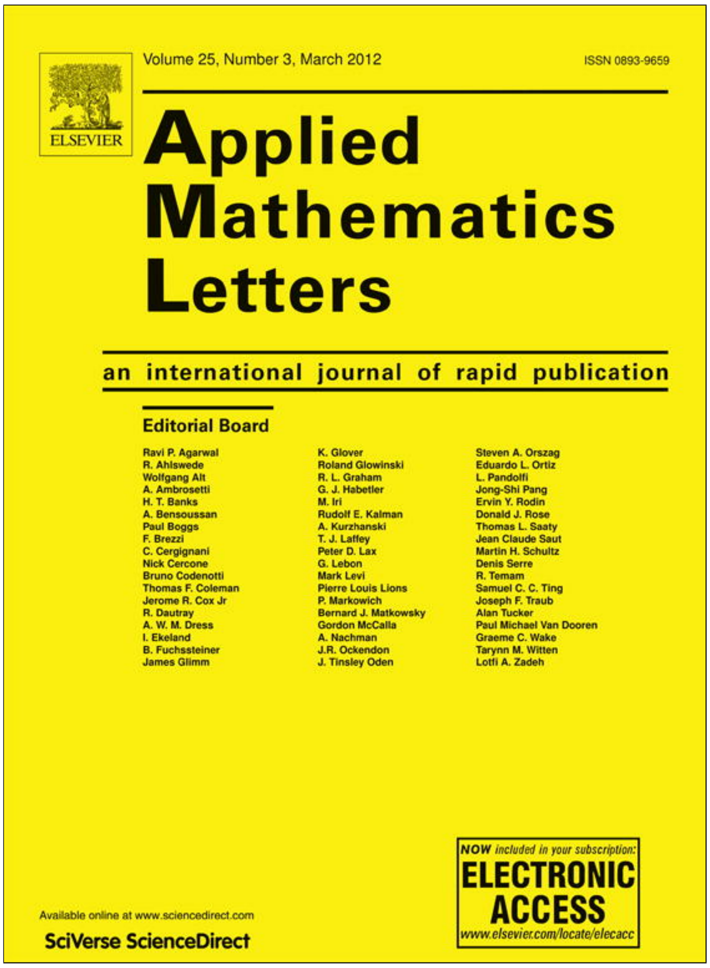
Volume 25, Number 3, March 2012
ISSN 0893-9659
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an international journal of rapid publication
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Applied Mathematics Letters

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Two unsolved problems in the stability theory of stochastic differential equations with delay

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ARTICLE INFO

Article history:
Received 18 December 2010
Received in revised form 10 September 2011
Accepted 3 October 2011

Keywords:
Unsolved problems
Stability
Stochastic differential equations
Delay
Necessary and sufficient condition

ABSTRACT

Two unsolved problems of the stability theory for stochastic differential equations with delay are offered for consideration.

1. Introduction

The problems of the stability theory for stochastic systems with delays are very popular topics for research. At the same time there are simply and clearly formulated problems with unknown solutions. In order to attract attention to such problems, one of them, for a stochastic difference equation with continuous time, is presented in [1] and two unsolved stability problems for stochastic differential equations with delay are proposed here.

Problem 1. Consider the linear stochastic differential equation with delays

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{m} B_i x(t-\tau_i) + \sigma x(t-h)\dot{w}(t), \tag{1.1}$$

where A, B_i, σ, $\tau_i > 0$, $h \geq 0$ are known constants, and $w(t)$ is the standard Wiener process.

It is known [2] that the necessary and sufficient condition for the asymptotic mean square stability of the zero solution of Eq. (1.1) can be represented in the form

$$G^{-1} > \frac{\sigma^2}{2}, \qquad G = \frac{2}{\pi}\int_0^\infty \frac{dt}{\left(A + \sum_{i=1}^{m} B_i \cos\tau_i t\right)^2 + \left(t + \sum_{i=1}^{m} B_i \sin\tau_i t\right)^2}. \tag{1.2}$$

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doi:10.1016/j.aml.2011.10.002

In the particular case $m = 1$, $B_1 = B$, $\tau_1 = \tau$, the integral (1.2) can be calculated (see [3, p.7–9]) using elementary functions and equals

$$G = \begin{cases} \dfrac{Bq^{-1}\sin(q\tau) - 1}{A + B\cos(q\tau)}, & B + |A| < 0, \quad q = \sqrt{B^2 - A^2}, \\ \dfrac{1 + |A|\tau}{2|A|}, & B = A < 0, \\ \dfrac{Bq^{-1}\sinh(q\tau) - 1}{A + B\cosh(q\tau)}, & A + |B| < 0, \quad q = \sqrt{A^2 - B^2}. \end{cases} \tag{1.3}$$

The problem is: *to calculate the integral (1.2) using elementary functions for $m \geq 2$, and in particular, for $m = 2$.*

Problem 2. From (1.2), (1.3) it follows that the zero solution of the differential equation with a constant delay $\dot{x}(t) = -bx(t - h)$ is asymptotically stable if and only if

$$0 < bh < \frac{\pi}{2}. \tag{2.1}$$

It is known also [4,5] that the zero solution of the differential equation with a varying delay $\dot{x}(t) = -bx(t - \tau(t))$ is asymptotically stable for an arbitrary delay $\tau(t)$ such that $\tau(t) \in [0, h]$ if and only if

$$0 < bh < \frac{3}{2}. \tag{2.2}$$

Consider the stochastic differential equation with a constant delay

$$\dot{x}(t) = -bx(t - h) + \sigma x(t)\dot{w}(t). \tag{2.3}$$

From (1.2) and (1.3) it follows that the zero solution of Eq. (2.3) is asymptotically mean square stable if and only if

$$0 < bh < \arcsin\frac{b^2 - p^2}{b^2 + p^2}, \qquad p = \frac{\sigma^2}{2}. \tag{2.4}$$

In the deterministic case ($\sigma = 0$) the condition (2.4) coincides with (2.1).

Consider the stochastic differential equation

$$\dot{x}(t) = -bx(t - \tau(t)) + \sigma x(t)\dot{w}(t) \tag{2.5}$$

with a varying delay $\tau(t)$ such that $\tau(t) \in [0, h]$.

The problem is: *to generalize the condition (2.2) for* Eq. (2.5).

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