21-240: Exam 1 Topics List

Definitions (this list is not necessarily complete! You should know all of the definitions in the sections we covered, these are the most important ones): linear equation, coefficients, system of linear equations,solution, solution set, equivalence of linear systems, consistent/inconsistent, echelon forms, leading entry, pivot position and column, basic and free variables, column vector, scalar and scalar multiple, linear combination, Span, homogeneous, trivial solution, non-trivial solution, networks/nodes/arcs, linearly independent and dependent, domain, codomain, range, image, mapping/function/transformation, what it means for a function to be linear, standard matrix for a linear transformation, onto (surjective), one to one (injective), bijective, diagonal entries of a matrix, main diagonal, diagonal matrix, zero matrix, matrix transpose, matrix inverse, invertible, singular/nonsingular, elementary matrix, identity matrix

Topics:

- The theorem that a linear system has 0, 1, or infinitely many solutions
- Matrix notation, coefficient matrix and augmented matrix
- Elementary row operations, row equivalence, echelon form and reduced row echelon form
- The theorem that if the augmented matrices of two linear systems are row equivalent, then the two systems have the same solution set
- Existence and uniqueness of solutions
- The row reduction algorithm
- Basic and free variables and parametric descriptions of solution sets
- The Existence and Uniqueness theorem
- Vector equations
- Span and whether or not a vector is in the span of a set of vectors
- The matrix equation $A\mathbf{x} = \mathbf{b}$
- The definition of matrix multiplication with a column vector

- $A\mathbf{x} = \mathbf{b}$ has a solution if and only if $\mathbf{b}$ is a linear combination of the columns of A.
- The equivalence of solutions to $A\mathbf{x} = \mathbf{b}$, the vector equation writing $\mathbf{b}$ as a linear combination of columns of A, and the system of linear equations with augmented matrix $[A\ \mathbf{b}]$
- The theorem about equivalence of solutions to $A\mathbf{x} = \mathbf{b}$ and A having a pivot position in every row
- A homogeneous equation $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution if and only if the equation has at least one free variable
- Parametric vector equations
- The theorem describing solutions of homogeneous and nonhomogeneous systems
- Writing a solution set in parametric vector form
- Applications of linear systems: network flows, balancing chemical equations, difference equations
- In a network flow: flow in = flow out
- Deciding whether a set of vectors is linearly independent or dependent
- Theorem: The columns of A are linearly independent if and only if $A\mathbf{x} = \mathbf{0}$ has only the trivial solution
- Sufficient conditions for vectors to be linearly dependent
- Showing that a transformation is linear
- Matrices as functions
- Theorem: if $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation, then there is a unique matrix A such that $A\mathbf{x} = T(\mathbf{x})$ for all $\mathbf{x}$
- Writing down what the matrix A corresponding to T is
- Showing a transformation is onto and/or one to one
- Theorem: T a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$, then T is one to one if and only if $T(\mathbf{0}) = \mathbf{0}$ has only the trivial solution
- Theorem: T a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ and A its standard matrix. T is onto if and only if the columns of A span $\mathbb{R}^m$ and T is one to one if and only if the columns of A are linearly independent
- Sums and scalar multiples of matrices
- Matrix multiplication

- Properties of matrix multiplication
- Properties of matrix transposition
- Theorem: If A is an invertible $n \times n$ matrix, then for all $\mathbf{b} \in \mathbf{R}^n$, the equation $A\mathbf{x} = \mathbf{b}$ has a unique solution $\mathbf{x} = A^{-1}\mathbf{b}$
- Theorem about invertible matrices and their products/transpositions
- Theorem: A square matrix A is invertible if and only if it is row equivalent to the identity matrix I. A sequence of elementary row operations transforming A to I also transforms I to A^{-1}
- Finding A^{-1} for a particular A
- The theorem with 12 equivalent statements about invertible matrices
- Invertible linear transformations and the theorem about their standard matrices