Math 301: Homework 11

Due Friday December 8 at noon on Canvas

1. Let G be the bipartite incidence graph of a projective plane of order q. Compute the eigenvalues of G (Hint: Let A be the adjacency matrix of G. Compute the eigenvalues of A^2).

2. The *Kneser graph* $\mathrm{KG}(n,k)$ is the graph whose vertex set is the k-element subsets of $[n]$ where A and B are adjacent if and only if they are disjoint. It is known that for $0 \leq j \leq k$, $\mathrm{KG}(n,k)$ has eigenvalue $(-1)^j\binom{n-k-j}{k-j}$ with multiplicity $\binom{n}{j} - \binom{n}{j-1}$ for $j > 0$ and 1 for $j = 0$. Use the Hoffman Ratio Bound to prove the Erdős-Ko-Rado theorem.

3. Let G be a graph on n vertices and let n_+ and n_- be the number of positive and negative eigenvalues of G respectively.

   (a) Use eigenvalue interlacing to prove the Cvetković Inertia Bound:
$$\alpha(G) \leq \min\{n - n_+, n - n-\}.$$

   (b) Give an example of a graph for which this bound is tight.

4. Let $m_r(G)$ denote the minimum number of complete at most r-partite graphs (each graph is complete k-partite with $2 \leq k \leq r$) that partition the edge set of G. Show that
$$m_r(G) \geq \frac{1}{r-1} n_-(G)$$
where n_- denotes the number of negative eigenvalues of G.