

# Math 301: Homework 10

Due Wednesday November 29 at noon on Canvas

1. Compute the spectrum of  $K_n$  and  $K_{m,n}$ .
2. Two vertices in a graph  $G$  are called *twins* if they have exactly the same neighborhood. Show that if a graph has a pair of twins then 0 is one of its eigenvalues.
3. Let  $G$  be a graph and let  $H$  be the graph obtained by adding an isolated vertex to  $G$ . Show that  $\text{spec}(H) = \text{spec}(G) \cup \{0\}$ .
4. Does the spectrum of a graph determine connectivity? ie, if I tell you the eigenvalues of a graph, can you tell me if it is connected? If not, can you construct two graphs with the same set of eigenvalues, one of which is connected and one of which is disconnected? Problems 1 and 3 may be useful.
5. Assume that  $G$  is a regular graph. Write the number of 5-cycles in  $G$  as a function of its eigenvalues.
6. Show that a graph is bipartite if and only if its spectrum is symmetric around 0.