Fall 2012 V2 Final

Solutions

1. a $\int_0^3 f(x)\,dx = 5$

$\int_0^3 g(x)\,dx = -2/3$

$$\boxed{\int_0^3 2f(x) + g(x)\,dx = 8}$$

b. $\int_{-2}^0 f(x)\,dx = -\int_0^2 f(x)\,dx = \boxed{0}$

c. $G(3) - G(0) = \int_0^3 g(x)\,dx = -2/3$

$$\boxed{G(3) = 1/3}$$

2. $$\int \frac{x^2+3x+1}{x}\,dx = \int x + 3 + \frac{1}{x} = \boxed{\frac{x^2}{2} + 3x + \ln|x| + C}$$

3. $$\frac{d}{dx}\int_3^{\sin(x)} e^{\sqrt{t}}\,dt = \boxed{e^{\sqrt{\sin(x)}} \cdot \cos(x)}$$

4. $$\int x^3 \ln x \, dx$$

$u = \ln x \qquad v' = x^3$

$du = \frac{1}{x} \qquad v = \frac{x^4}{4}$

$$= \frac{x^4 \ln x}{4} - \int \frac{x^3}{4}\,dx$$

$$= \boxed{\frac{x^4 \ln x}{4} - \frac{x^4}{16} + C}$$

5. $$\int_0^1 \frac{x}{\sqrt{4+x^2}}\,dx$$

$$u = 4+x^2 \qquad \frac{du}{dx} = 2x$$

$$= \int_4^5 \frac{\frac{1}{2}\,du}{\sqrt{u}} = u^{1/2}\Big|_4^5$$

$$= \sqrt{4+x^2}\Big|_0^1$$

$$= \boxed{\sqrt{5}-2}$$

6. $$\frac{x}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$x = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$$

$x=-1$: $\quad C = 1/2 \qquad x = 1$: $\quad A = 1/4$

Constant term: $\quad A - C - B = 0$

$B = -1/4$

$$\boxed{\int \frac{x}{(x-1)(x+1)^2} = \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| - \frac{1}{2}(x+1)^{-1} + C}$$

7 a. $\displaystyle\int_2^5 \frac{dx}{\sqrt{x-2}} = \lim_{r\to 2^+} \int_r^5 \frac{dx}{\sqrt{x-2}}$

$$= \lim_{r\to 2^+} 2\sqrt{x-2}\,\Big|_r^5$$

$$= \boxed{2\sqrt{3}}$$

b. $\displaystyle\int_1^\infty \frac{\ln x}{x^3+1} \le \int_1^\infty \frac{x}{x^3+1} \le \int_1^\infty \frac{x}{x^3}\,dx$

$= \displaystyle\int_1^\infty \frac{dx}{x^2}$ which converges by p-test.

Therefore, $\displaystyle\int_1^\infty \frac{\ln x}{x^3+1}\,dx$ converges by comparison test

8.

$$\text{Volume} = \int_0^{\pi/4} \pi\left(\sqrt{\tan x \sec^2 x}\right)^2 dx$$

$$= \int_0^{\pi/4} \pi \tan x \sec^2 x \, dx$$

$(u = \sec x)$

$$= \frac{\pi}{2} \sec^2 x \Big|_0^{\pi/4}$$

$$= \frac{\pi}{2}(2) - \frac{\pi}{2}(1) = \boxed{\pi/2}$$

9. $y = e^{kx}$ $\quad \frac{dy}{dx} = ke^{kx}$ $\quad \frac{d^2y}{dx^2} = k^2 e^{kx}$

$k^2 e^{kx} = 4e^{kx}$ iff $\boxed{k = \pm 2}$

10. $$\int \frac{dy}{y-2} = \int -t\,dt$$

$$\ln|y-2| = -\frac{t^2}{2} + C$$

$$y - 2 = Ae^{-t^2/2}$$

$$3 = Ae^0$$

$$\boxed{y = 3e^{-t^2/2} + 2}$$