Name: _____ **PID:**_____

Circle your section: A01 (11am-12pm) or A02 (12pm-1pm)

MATH 10B: PRACTICE FINAL EXAM

July 29th, 2012

Do not turn the page until instructed to begin.

Turn off and put away your cell phone.

No calculators or any other devices are allowed.
You may use one 8.5×11 page of handwritten notes, but no other assistance.
Read each question carefully, answer each question completely, & show all of your work.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.
Good luck!

#	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
Σ	80	

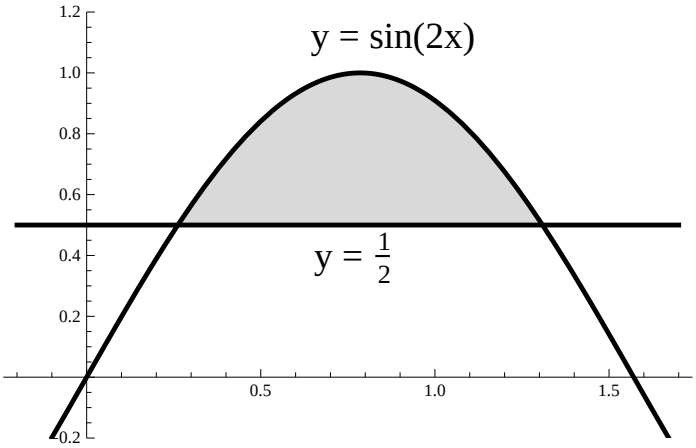
1. Take the following indefinite integrals.

(a) (5 points) $\displaystyle\int \frac{(2-x)^2}{x}\,dx.$ (hint: FOIL and separate terms)

(b) (5 points) $\displaystyle\int \frac{x}{(2-x)^2}\,dx.$ (hint: u-substitution)

2. (10 points) A company has a continuous income stream of $P(t) = 10 + t$ million dollars per year. Assuming this income is directly deposited in an account making 5% interest per year, how much money will the company have in 2 years?

3. (10 points) Find the shaded area between the curves depicted below.

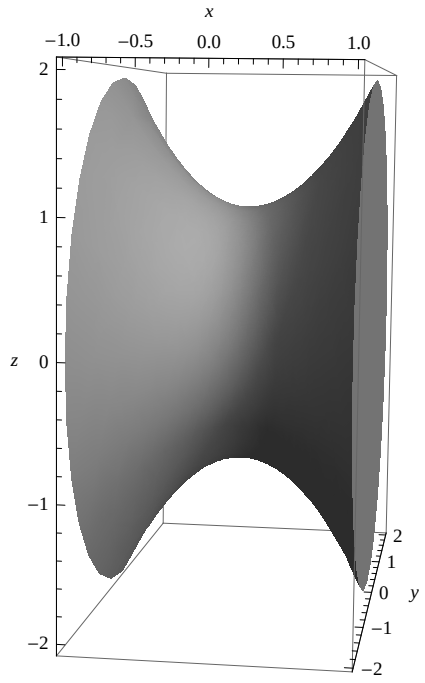


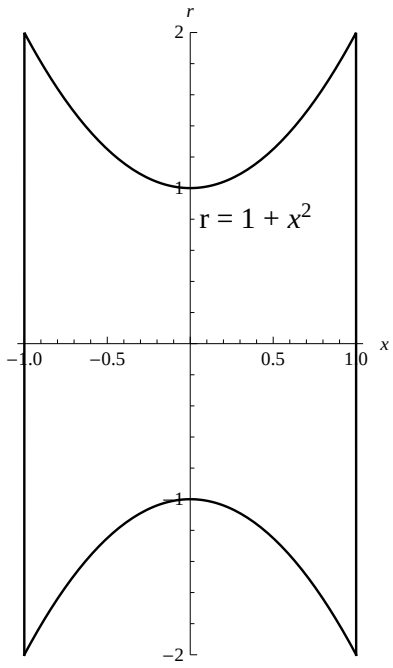
4. Find more indefinite integrals.

(a) (5 points) $\displaystyle\int \frac{1}{1+9x^2}\,dx.$ (hint: trig. substitution)

(b) (5 points) $\displaystyle\int \frac{18x}{1+9x^2}\,dx.$ (hint: u-substitution)

5. (10 points) Find the volume of the spool depicted below with radius given by the curve $r = 1 + x^2$ for $-1 \leq x \leq 1$.





6. Take the following indefinite integrals.

(a) (5 points) $\int \frac{\ln(x)}{x}\, dx.$ (hint: u-substitution)

(b) (5 points) $\int \frac{2}{x^2 - x}\, dx.$ (hint: partial fractions)

7. Solve the following initial value problem.

$$\frac{dy}{dx} = x\sin(x) \qquad\qquad y(0) = 1.$$

8. (10 points) You're drinking coffee while working on a practice final, but you're so concentrated on the questions that you forget to continue drinking. Your coffee starts at $150°$F and the temperature of the room is $75°$F. According to Newton's Law of Cooling, the coffee's temperature $y(t)$ (in degrees Fahrenheit) as a function of time (in minutes) is given by the differential equation

$$\frac{dy}{dt} = \frac{75 - y}{50} \qquad\qquad y(0) = 150,$$

How long until the coffee reaches $100°$F?

TABLE OF INTEGRALS

BASIC FUNCTIONS

1. $\displaystyle\int x^n\,dx = \frac{1}{n+1}\,x^{n+1} + C \qquad (\text{if } n \neq 1)$
2. $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$
3. $\displaystyle\int a^x\,dx = \frac{1}{\ln(a)}\,a^x + C \qquad (\text{if } a > 0)$
4. $\displaystyle\int \ln(x)\,dx = x\,\ln(x) - x + C$
5. $\displaystyle\int \sin(x)\,dx = -\cos(x) + C$
6. $\displaystyle\int \cos(x)\,dx = \sin(x) + C$
7. $\displaystyle\int \tan(x)\,dx = -\ln|\cos(x)| + C$

PRODUCTS OF e^x, $\cos(x)$, $\sin(x)$

8. $\displaystyle\int e^{ax}\sin(bx)\,dx = \frac{1}{a^2+b^2}\,e^{ax}\left[a\sin(bx) - b\cos(bx)\right] + C$
9. $\displaystyle\int e^{ax}\cos(bx)\,dx = \frac{1}{a^2+b^2}\,e^{ax}\left[a\cos(bx) + b\sin(bx)\right] + C$
10. $\displaystyle\int \sin(ax)\sin(bx)\,dx = \frac{1}{b^2-a^2}\left[a\cos(ax)\sin(bx) - b\sin(ax)\cos(bx)\right] + C \quad (\text{if } a \neq b)$
11. $\displaystyle\int \cos(ax)\cos(bx)\,dx = \frac{1}{b^2-a^2}\left[b\cos(ax)\sin(bx) - a\sin(ax)\cos(bx)\right] + C \quad (\text{if } a \neq b)$
12. $\displaystyle\int \sin(ax)\cos(bx)\,dx = \frac{1}{b^2-a^2}\left[b\sin(ax)\sin(bx) + a\cos(ax)\cos(bx)\right] + C \quad (\text{if } a \neq b)$

PRODUCT OF POLYNOMIAL $p(x)$ WITH $\ln(x)$, e^x, $\cos(x)$, $\sin(x)$

13. $\displaystyle\int x^n\ln(x)\,dx = \frac{1}{n+1}\,x^{n+1}\ln(x) - \frac{1}{(n+1)^2}x^{n+1} + C \qquad (\text{if } n \neq -1)$
14. $\displaystyle\int p(x)e^{ax}\,dx = \frac{1}{a}p(x)e^{ax} - \frac{1}{a}\int p'(x)\,e^{ax}\,dx$
$\displaystyle= \frac{1}{a}p(x)e^{ax} - \frac{1}{a^2}p'(x)e^{ax} + \frac{1}{a^3}p''(x)\,e^{ax} - \cdots \qquad (+-+-+-\ldots)$
15. $\displaystyle\int p(x)\sin(ax)\,dx = -\frac{1}{a}p(x)\cos(ax) + \frac{1}{a}\int p'(x)\cos(ax)\,dx$
$\displaystyle= -\frac{1}{a}p(x)\cos(ax) + \frac{1}{a^2}p'(x)\sin(ax) + \frac{1}{a^3}p''(x)\cos(ax) - \cdots \quad (-++--++\ldots)$
16. $\displaystyle\int p(x)\cos(ax)\,dx = \frac{1}{a}p(x)\sin(ax) - \frac{1}{a}\int p'(x)\sin(ax)\,dx$
$\displaystyle= \frac{1}{a}p(x)\sin(ax) + \frac{1}{a^2}p'(x)\cos(ax) - \frac{1}{a^3}p''(x)\sin(ax) - \cdots \quad (++--++\ldots)$

INTEGER POWERS OF $\sin(x)$, $\cos(x)$

17. $\displaystyle\int \sin^n(x)\,dx = -\frac{1}{n}\sin^{n-1}(x)\,\cos(x) + \frac{n-1}{n}\int \sin^{n-2}(x)\,dx + C \qquad (\text{if } n > 0)$

18. $\displaystyle\int \cos^n(x)\,dx = \frac{1}{n}\cos^{n-1}(x)\,\sin(x) + \frac{n-1}{n}\int \cos^{n-2}(x)\,dx + C \qquad (\text{if } n > 0)$

19. $\displaystyle\int \frac{1}{\sin^m(x)}\,dx = \frac{-1}{m-1}\,\frac{\cos(x)}{\sin^{m-1}(x)} + \frac{m-2}{m-1}\int \frac{1}{\sin^{m-2}(x)}\,dx \qquad (\text{if } m > 1)$

20. $\displaystyle\int \frac{1}{\sin(x)}\,dx = \frac{1}{2}\ln\left|\frac{\cos(x)-1}{\cos(x)+1}\right| + C$

21. $\displaystyle\int \frac{1}{\cos^m(x)}\,dx = \frac{1}{m-1}\,\frac{\sin(x)}{\cos^{m-1}(x)} + \frac{m-2}{m-1}\int \frac{1}{\cos^{m-2}(x)}\,dx \qquad (\text{if } m > 1)$

22. $\displaystyle\int \frac{1}{\cos(x)}\,dx = \frac{1}{2}\ln\left|\frac{\sin(x)+1}{\cos(x)-1}\right| + C$

23. $\displaystyle\int \sin^m(x)\cos^n(x)\,dx$

If m is odd, let $w = \cos(x)$. If n is odd, let $w = \sin(x)$. If both m and n are even and nonnegative, convert all to $\sin(x)$ or all to $\cos(x)$ (using $\cos^2(x) + \sin^2(x) = 1$), and use 17 or 18. If m and n are even and one of them is negative, convert to whichever function is in the denominator and use 19 or 21. If both m and n are even and negative, substitute $w = \tan(x)$.

QUADRATIC IN THE DENOMINATOR

24. $\displaystyle\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$

25. $\displaystyle\int \frac{bx+c}{x^2+a^2}\,dx = \frac{b}{2}\ln|x^2+a^2| + \frac{c}{a}\arctan\left(\frac{x}{a}\right) + C \qquad (\text{if } a \neq 0)$

26. $\displaystyle\int \frac{1}{(x-a)(x-b)}\,dx = \frac{1}{a-b}(\ln|x-a| - \ln|x-b|) + C \qquad (\text{if } a \neq b)$

27. $\displaystyle\int \frac{cx+d}{(x-a)(x-b)}\,dx = \frac{1}{a-b}[(ac+d)\ln|x-a| - (bc+d)\ln|x-b|] + C \qquad (\text{if } a \neq b)$

INTEGRANDS INVOLVING $\sqrt{a^2+x^2}$, $\sqrt{a^2-x^2}$, $\sqrt{x^2-a^2}$, $\quad a > 0$

28. $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right) + C$

29. $\displaystyle\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

30. $\displaystyle\int \sqrt{a^2 \pm x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2 \pm x^2} + a^2\int \frac{1}{\sqrt{a^2 \pm x^2}}\,dx\right) + C$

31. $\displaystyle\int \sqrt{x^2-a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2-a^2} - a^2\int \frac{1}{\sqrt{x^2-a^2}}\,dx\right) + C$