parametrizing surfaces:
we need 2 variables.

eg: sphere of radius 5

$$X = 5\sin(u)\cos(v)$$
$$Y = 5\sin(u)\sin(v)$$
$$Z = 5\cos(u)$$

$$0 \le u \le \pi$$
$$0 \le v \le 2\pi$$

in 2D, parameterization was "what are the coordinates of the fly at time t?"

in 3D, we have run out of dimensions. I can't really expain it like that.

Surface area

Formula: $$A(S) = \iint_D |\vec{r}_u \times \vec{r}_v| \, du\, dv$$

eg: sphere: $\vec{r} = \langle 5\sin(u)\cos(v),\ 5\sin(u)\sin(v),\ 5\cos(u) \rangle$

$$\vec{r}_u = \langle 5\cos(u)\cos(v),\ 5\cos(u)\sin(v),\ -5\sin(u) \rangle$$

$$\vec{r}_v = \langle -5\sin(u)\sin(v),\ 5\sin(u)\cos(v),\ 0 \rangle$$

$$\vec{r}_u \times \vec{r}_v = \langle 0 + 25\sin^2 u \cos v,\ -(0 - 25\sin^2 u \sin v),\ 25 \sin u \cos^2 v \cos u + 25 \sin u \sin^2 v \cos u \rangle$$

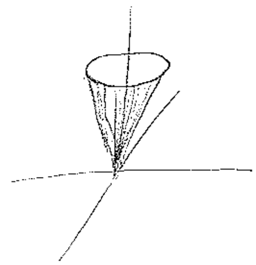
$$= 25 \langle \sin^2 u \cos v,\ \sin^2 u \sin v,\ 25 \sin u \cos u \rangle$$

$$|\vec{r}_u \times \vec{r}_v| = 25 \sin u$$

$$\int_0^{2\pi}\int_0^{\pi} 25 \sin u \, du\, dv = \int_0^{2\pi} -\left[25\cos u\right]_0^{\pi} dv = \int_0^{2\pi} 2\cdot 25 = 2\pi \cdot 2 \cdot 25 = 4\pi(5)^2$$

Surface area of a cone of height 7 and radius 5.

Curved part of the cone:



in cylindrical coordinates, the equation is $\frac{r}{5} = \frac{z}{7}$

$$X = r\cos\theta = \frac{5}{7} z \cos\theta$$
$$Y = r\sin\theta = \frac{5}{7} z \cos\theta$$

We use the parameterization

$$X = \frac{5}{7} u \cos(v) \qquad 0 \le v \le 2\pi$$
$$Y = \frac{5}{7} u \sin(v) \qquad 0 \le u \le 7$$
$$Z = u$$

$$\vec{r}_u = \left\langle \frac{5}{7}\cos(v),\ \frac{5}{7}\sin(v),\ 1 \right\rangle$$

$$\vec{r}_v = \left\langle -\frac{5}{7}u\sin(v),\ \frac{5}{7}u\cos(v),\ 0 \right\rangle$$

$$\vec{r}_u \times \vec{r}_v = \left\langle 0 - \frac{5}{7}u\cos(v),\ -\left(0 + \frac{5}{7}u\sin(v)\right),\ \frac{25}{49}u\cos^2 v + \frac{25}{49}u\sin^2 v \right\rangle$$

$$= \left\langle -\frac{5}{7}u\cos(v),\ -\frac{5}{7}u\sin(v),\ \frac{25}{49}u \right\rangle$$

$$|\vec{r}_u \times \vec{r}_v| = u\sqrt{\frac{25}{49} + \left(\frac{25}{49}\right)^2}$$

$$A(S) = \int_0^{2\pi}\int_0^7 u\sqrt{\frac{25}{49}\left(\frac{25+49}{49}\right)}\,du\,dv = \int_0^{2\pi}\left[\frac{u^2}{2}\,\frac{5}{49}\sqrt{74}\right]_0^7 = \int_0^{2\pi}\frac{5}{2}\sqrt{74}$$

$$= 2\pi\cdot\frac{5}{2}\sqrt{74} = 5\pi\sqrt{74} = 5\pi\sqrt{5^2+7^2}$$

Adding in the circle to close the cone gives a surface area of

$$5\pi\sqrt{5^2+7^2} + 5^2\cdot\pi$$