Green's Thm: $\oint_C P\,dx + Q\,dy = \iint_D (Q_x - P_y)\,dA.$

if $\vec{F} = \langle P, Q \rangle$

$dr = \langle dx, dy \rangle$ $\quad \oint_C P\,dx + Q\,dy = \oint_C \vec{F}\cdot dr = \oint_C \vec{F} \cdot \frac{dr}{ds}\,ds = \oint_C \vec{F}\cdot\hat{T}\,ds.$

$\iint_D (Q_x - P_y)\,dA = \iint_D \text{curl}(\vec{F})\,dA.$

if $\vec{F} = \langle Q, -P \rangle$

$\hat{n}$ = vector normal to the curve (unit vector)

at any point (x,y) $\quad \hat{n} = \langle y'(t), -x'(t) \rangle \cdot \frac{1}{\sqrt{(x')^2 + (y')^2}}$

$$\vec{F}\cdot\hat{n} = \frac{[(Q)(y') + (P)(x')]}{\sqrt{(x')^2 + (y')^2}}$$

$$= \frac{P\frac{dx}{dt} + Q\frac{dy}{dt}}{\left|\frac{ds}{dt}\right|}$$

so $\quad \oint_C P\,dx + Q\,dy = \oint_C \left(P\frac{dx}{ds} + Q\frac{dy}{ds}\right)ds = \oint_C \vec{F}\cdot\hat{n}\,ds$

$$\iint_D (Q_x - P_y)\,dA = \iint_D \text{div}(\vec{F})\,dA$$

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so green's Thm in vector forms:

① $\oint_C \vec{F}\cdot\hat{T}\,ds = \iint_D \text{curl}(\vec{F})\,dA$

② $\oint_C \vec{F}\cdot\hat{n}\,ds = \iint_D \text{div}(\vec{F})\,dA.$