Spherical Coordinates

Concept:
ρ is the distance to the origin
θ is the angle to the xz plane (as in cylindrical)
ϕ is the angle to the z axis

Computation:
$\rho = \sqrt{x^2 + y^2 + z^2}$
$\tan\theta = \frac{y}{x}$
$\tan\phi = \frac{\sqrt{x^2+y^2}}{z}$, $0 \le \phi \le \pi$

$x = \rho\cos\theta\sin\phi$
$y = \rho\sin\theta\sin\phi$
$z = \rho\cos\phi$

$dV = \rho^2\sin\phi\ d\rho\ d\theta\ d\phi$

Problems:

1. P is given in spherical coordinates as (2, $\pi/3$, $\pi/4$). Plot P and convert it to rectangular coordinates.
2. P is given in rectangular coordinates as (0, -1, -1). Convert this to spherical coordinates.
3. Indentiry the surface $\rho = 2\cos\phi$
4. Sketch the solid described by
   $-\pi/2 \le \theta \le \pi/2$
   $0 \le \phi \le \pi/6$
   $0 \le \rho \le \sec\phi$
5. Use spherical coordinates to find the volume of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$.
6. Find the volume of the solid bounded below by the cone $z = \sqrt{x^2 + y^2}$, above by the plane $z = 4$, and on its sides by the cylinder $x^2 + y^2 = 4$.
   (a) Using rectangular coordinates
   (b) Using cylindrical coordinates
   (c) Using spherical coordinates

Answers:

1. $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{6}}{2}, \sqrt{2}\right)$
2. $\left(\sqrt{2}, 3\pi/2, 3\pi/4\right)$
3. Sphere centered at (0,0,1) with radius 1
4. Cone with $\phi = \pi/6$ and a flat top at $z = 1$.
5. $\pi/8$
6. $64\pi/3$