Integrals

We are about to start doing lots and lots of integrals. We won't go over this in class or recitation, (we just don't have time) so if you don't remember how to integrate using u-substitution, integation by parts, partial fractions, etc., now is a good time to brush up.

Section 5.5: Use u-substitution to integrate.

29. $\int \frac{e^x + 1}{e^x} dx$

43. $\int_1^2 x\sqrt{x-1} dx$

Section 6.1: Use integration by parts to integrate.

3. $\int x \cos 5x dx$

17. $\int_1^2 \frac{\ln x}{x^2}$

Section 6.3: Use partial fractions to integrate.

9. $\int \frac{x-9}{(x+5)(x-2)} dx$

19. $\int \frac{1}{(x+5)^2(x-1)}$

Mixed: Evaluate the integral

1. $\int \arctan 4t dt$

2. $\int \frac{x-1}{x^2+3x+2} dx$

3. $\int \frac{6}{(2x+3)^3} dx$

4. $\int xe^x dx$

5. $\int \cos t \sin^5 t dt$

6. $\int \frac{x+3}{x-6x+8} dx$

Double integrals over rectangles

Concept: Find the volume of the region under a surface (over a regtangular domain)
Computation: Integrate twice. Make sure you do it in the right order.

Evaluate the double integral

1. $\int_1^2 \int_{-1}^1 xydxdy$
2. $\int_0^1 \int_{1/2}^2 \frac{x}{y} dydx$
3. $\int_1^5 \int_0^\pi xy \sin x dxdy$

Double integrals over other domains

Concept: Find the volume of a region under a surface (over an arbitrary domain)
Computation: Integrate twice. The bounds might be functions

Evaluate the double integrals

4. $\int_0^1 \int_0^{x^2} (x+2y)dydx$
5. $\int_0^1 \int_y^{e^y} \sqrt{x}dxdy$
6. $\int_0^{\pi/2} \int_0^{\cos\theta} e^{\sin\theta} drd\theta$

Sketch the domains of the following integrals

7. $\int_0^4 \int_0^{\sqrt{x}} f(x,y)dydx$
8. $\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} f(x,y)dydx$
9. $\int_1^2 \int_0^{\ln x} f(x,y)dydx$

Evaluate

10. $\iint_D x^3y^2 dA$ where $D = \{(x, y) — 0 \leq x \leq 2, -x \leq y \leq x\}$
11. $\iint_D x \cos y dA$ where D is bounded by $y = 0$, $y = x^2$, $x = 1$.
12. $\iint_D 2xy dA$ where D is the triangular region with vertices (0,0), (1,2), and (0,3).

Find the double integral by reversing the order of integration

13. $\int_0^1 \int_{3y}^3 e^{x^2} dxdy$
14. $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3 + 1} dxdy$
15. $\int_0^1 \int_{\arcsin y}^{\pi/2} \cos x \sqrt{1 + \cos^2 x} dxdy$

Find the volume of the given solid

16. Under the plane $x + 2y - z = 0$ and above the region bounded by $y = x$ and $y = x^4$
17. Bounded by the cylinder $x^2 + y^2 = 1$, the planes $y = z$, $x = 0$, and $z = 0$ in the first octant
18. The solid enclosed by the parabolic cylinder $y = x^2$ and the planes $z = 3y$, $z = 2+y$. (Subtract two volumes for this one).