Directional Derivatives

Find the Derivative of f at point P in the direction of $\mathbf{v}$.

1. $z = x\sin y + x^2$, $P(0, \frac{\pi}{4}, 0)$, $\mathbf{v} = \langle -\frac{8}{17}, \frac{15}{17} \rangle$
2. $z^2 = x^2 - y^3$, $P(3, 2, 1)$, $\mathbf{v} = \langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \rangle$
3. $z = \ln(xy^2)$, $P(1, e, 2)$, $\mathbf{v} = \langle 1 - 3 \rangle$
4. $e^{yz} = zx - \cos(xy)$, $P(1, 0, 1)$, $\mathbf{v} = \langle -5, 12 \rangle$

Local Mins & Maxs

Concept: Find the local mins and maxs of the graph
Computation: First find places where f_x and f_y are both 0. Then Check $D = (f_{xx})(f_{yy}) - (f_{xy})^2$. If $D > 0$, it's a local min or max. If $D < 0$ it's a saddle point.

Find and classify the critical points of the following surfaces

5. $z = 9 - 2x + 4y - x^2 - 4y^2$
6. $z = (1 + xy)(x + y)$
7. $z = e^x \cos y$
8. $z = x \sin y$
9. $z = (x^2 + y^2)e^{y^2 - x^2}$

Find the absolute maximum and minimum values of f on the set D.

10. $f(x, y) = 1 + 4x - 5y$,
    D is the closed triangular region with vertices (1,0), (5,0), and (1,4)
11. $f(x, y) = x^2 + y^2 + x^2y + 4$
    $D = \{(x, y) : |x| \le 1, |y| \le 1\}$
12. $f(x, y) = x^4 + y^4 - 4xy + 2$
    $D = \{(x, y) : 0 \le x \le 3, 0 \le y \le 2$

13. Find the shortest distance from the point (2, 1, -1) to the plane $x + y - z = 1$
14. Find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point (4,2,0)
15. Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $x + 2y + 3z = 6$.
16. Find the dimensions of a rectangular box of maximum volume such that the sum of the lengths of the edges is a constant c.