Chain Rule

Concept: Given a curve in space, where x and y are given in terms of t, but z is expressed in terms of x and y, how do we find dz/dt without the pain of substituting for x and y?
Computation: $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$

Find $\frac{dz}{dt}$ for the following functions.

1. $z = \sin x \cos y$, $x = \pi t$, $y = \sqrt{t}$
2. $z = e^{x/y}$, $x = 1 - t$, $y = 1 + 2t$
3. $z = 4x^2y - 2y^5$, $x = \sin t$, $y = \cos t$
4. $z = \frac{x}{y} + xy$ $x = e^t$, $y = \ln t$

Find $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$ for the following functions.

5. $z = x^2 + xy + y^2$, $x = s + t$, $y = st$
6. $z = e^r \cos\theta$, $r = st$, $\theta = \sqrt{s^2 + t^2}$
7. $z = \sin\alpha \tan\beta$, $\alpha = 3s + t$, $\beta = s - t$

Find $\frac{dz}{dt}$ at the specified point.

8. $z = x^3y + \sin y$, $x = e^t$, $y = 3t$, $t = \pi/4$
9. $z = \tan(x + y)$, $x = \ln(t)$, $y = t^2$, $t = 1$
10. $z = x^y$, $x = \sin(t)$, $y = 2t$, $t = 4$

Directional Derivatives

Concept: Partial derivatives are slope in the **i** and **j** directions, but what about the slopes in all the other directions?
Computation: If $\mathbf{u}$ is a unit vector, $D_{\mathbf{u}}f(x, y) = \mathbf{u} \cdot \nabla f$, where $\nabla f = \langle f_x, f_y\rangle$.

Find ∇f and the rate of change of f at P in the direction of the given vector.

11. $f(x, y) = 5xy^2 - 4x^3y$, $P(1, 2)$, $\mathbf{u} = \langle \frac{5}{13}, \frac{12}{13}\rangle$
12. $f(x, y) = y \ln x$, $P(1, -3)$, $\mathbf{u} = \langle -\frac{4}{5}, \frac{3}{5}\rangle$
13. $f(x, y, z) = xe^{2yz}$, $P(3, 0, 2)$, $\mathbf{u} = \langle \frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\rangle$
14. $f(x, y) = 1 + 2x\sqrt{y}$, $P(3, 4)$, $\mathbf{v} = \langle 4, -3\rangle$
15. $f(x, y) = x^2e^y$, $P(2, 0)$, $\mathbf{v} = \langle 1, 1\rangle$
16. $f(x, y, z) = \frac{x}{y + z}$, $P(4, 1, 1)$, $\mathbf{v} = \langle 1, 2, 3\rangle$