ANSWERS 3/1/11

1. $$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$(\cos x \cos y)\,\pi - (\sin x \sin y)\frac{1}{2\sqrt{t}}$$

2. $$\frac{1}{y}e^{x/y}\cdot 1 - \frac{x}{y^2}e^{x/y}\cdot 2$$

3. $$8xy\cdot\cos t - (4x^2-10y)\cdot\sin t$$

4. $$\left(\frac{1}{y}+y\right)e^t + \left(-\frac{x}{y^2}+x\right)\frac{1}{t}$$

5. $$\frac{\partial z}{\partial t} = (2x+y)1 + (x+2y)s$$

$$\frac{\partial z}{\partial s} = (2x+y)1 + (x+2y)t$$

6. $$\frac{\partial z}{\partial t} = (e^r\cos\theta)s - (e^r\sin\theta)\frac{2t}{2\sqrt{s^2+t^2}}$$

$$\frac{\partial z}{\partial s} = (e^r\cos\theta)t - (e^r\sin\theta)\left(\frac{2s}{2\sqrt{s^2+t^2}}\right)$$

7. $$\frac{\partial z}{\partial t} = (\cos\alpha\tan\beta)1 + (\sin\alpha\sec^2\beta)(-1)$$

$$\frac{\partial z}{\partial s} = (\cos\alpha\tan\beta)3 + (\sin\alpha\sec^2\beta)(1)$$

8. $$\frac{dz}{dt} = 3x^2ye^t + (x^3+\cos y)3 \qquad x=e^{\pi/4},\quad y=\frac{3}{4}\pi$$

$$z'\left(\frac{\pi}{4}\right) = 3e^{2\pi/4}\cdot\frac{3}{4}\pi e^{\pi/4} + 3\left(e^{3\pi/4} + \left(-\frac{\sqrt{2}}{2}\right)\right)$$

$$= \frac{9}{4}\pi e^{3\pi/4} + 3e^{3\pi/4} - \frac{3\sqrt{2}}{2}$$

9. $\frac{dz}{dt} = \sec^2(x+y) \cdot \frac{1}{t} + \sec^2(x+y)\, 2t \qquad t=1,\ x=0,\ y=1$

$z'(1) = \sec^2(1)\big(1+2\big) = 3\sec^2(1)$

10. $\frac{dz}{dt} = \left(y\,x^{y-1}\right)\left(\cos t\right) + \left(\ln x\right)\left(x^{y}\right)\left(2\right) \qquad t=4,\ x=\sin 4,\ y=8$

$z'(4) = 8\sin^7 4 \cdot \cos 4 + 2\ln(\sin 4)\cdot \sin^8 4$

11. $\nabla f = \langle 5y^2 - 12x^2y,\ 10xy - 4x^3 \rangle = \langle 20-24,\ 20-4 \rangle = \langle -4, 16 \rangle$

$D_u f = \frac{-20}{13} + \frac{(12)(16)}{13} = \frac{172}{13}$

2. $\nabla f = \langle \frac{y}{x},\ \ln x \rangle = \langle -3, 0 \rangle$

$D_u f = \frac{12}{5}$

13. $\nabla f = \langle e^{2yz},\ 2xz\,e^{2yz},\ 2xye^{2yz} \rangle = \langle 1, 12, 0 \rangle$

$D_u f = \frac{2}{3} - \frac{24}{3} = -\frac{22}{3}$

14. $\hat{v} = \langle \frac{4}{5}, -\frac{3}{5} \rangle, \quad \nabla f = \langle 2\sqrt{y},\ \frac{2x}{2\sqrt{y}} \rangle = \langle 4, \frac{3}{2} \rangle$

$D_{\hat{v}} f = \frac{16}{5} - \frac{9}{10} = \frac{23}{10}$

5. $\hat{v} = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle \quad \nabla f = \langle 2xe^y,\ x^2e^y \rangle = \langle 4, 4 \rangle$

$D_{\hat{v}} f = \frac{4}{\sqrt{2}} + \frac{4}{\sqrt{2}} = 4\sqrt{2}$

6. $\hat{v} = \langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \rangle \quad \nabla f = \langle \frac{1}{y+z},\ -\frac{x}{(y+z)^2},\ -\frac{x}{(y+z)^2} \rangle = \langle \frac{1}{2}, -1, -1 \rangle$

$D_{\hat{v}} f = \frac{1/2}{\sqrt{14}} - \frac{2}{\sqrt{14}} - \frac{3}{\sqrt{14}} = -\frac{9}{2\sqrt{14}}$