

How many times do the following pairs of curves intersect? Find all points of intersection, and the angles at which they intersect.

1. $\mathbf{r}_1(t) = \langle t^2, t + 1, 5 \rangle$
 $\mathbf{r}_2(s) = \langle 9, s^2, s + 7 \rangle$

2. $\mathbf{r}_1(t) = \langle t^2, 6, -t \rangle$
 $\mathbf{r}_2(s) = \langle s, \cos(2\pi s), \sqrt{s} \rangle$

3. $\mathbf{r}_1(t) = \langle t, t^3 + 1, 0 \rangle$
 $\mathbf{r}_2(s) = \langle s - 1, s, \sin(\pi s) \rangle$

Find the arclength of the following curves

4. $\mathbf{f}(t) = \langle -\frac{1}{2}t^2, \frac{1}{15}(10t)^{3/2}, 5t \rangle, -2 \leq t \leq 2$

5. $\mathbf{f}(t) = \langle \ln(t), -\sqrt{2} \cdot t, \frac{1}{2}t^2 \rangle, 1 \leq t \leq e$

6. $\mathbf{f}(t) = \langle -e^{2t}, \frac{1}{2}e^{2t}, -e^{2t} \rangle, 0 \leq t \leq 1$

7. We have seen that there are multiple ways to parameterize a function. For example, two sets of equations can describe coinciding lines even though they look different. Does the arclength of a function depend on the parameterization we use to describe it?

- Reparameterize the function in problem 6 using $u = e^{2t}$
- What are the corresponding bounds?
- Find the arclength of this new curve.
- Is your answer the same as in problem 6?

8. A bird is flying in the air following the curve

$$\mathbf{f}(t) = \langle 4 \cos t, 4 \sin t, 20 - 3t \rangle \text{ from } t = 0 \text{ to } t = 2\pi$$

- Describe this curve in words
- The sun is directly overhead, so the bird's shadow is directly beneath him. What shape does the shadow trace?
- Using geometry, calculate the distance the shadow travels.
- How far above the ground is the bird at the start and end? How much does his height change?
- Use parts (c) and (d) to calculate the distance flown by the bird.
- Use calculus to calculate the distance flown by the bird.