1. $t^2 = 9$

$t + 1 = s^2$

$5 = s + 7$

$s = -2$

$t = (-2)^2 - 1 = 3$

$3^2 = 9$ ✓

the curves intersect once at $(9, (-2)^2, -2+7) = (9, 4, 5)$.

$\vec{r}_1{}'(t) = \langle 2t, 1, 0 \rangle$

$\vec{r}_1{}'(3) = \langle 6, 1, 0 \rangle$

$\hat{T}_1 = \frac{1}{\sqrt{37}} \langle 6, 1, 0 \rangle$

$\vec{r}_2{}'(s) = \langle 0, 2s, 1 \rangle$

$\vec{r}_2{}'(-2) = \langle 0, -4, 1 \rangle$

$\hat{T}_2 = \frac{1}{\sqrt{17}} \langle 0, -4, 1 \rangle$

$\cos\theta = \hat{T}_1 \cdot \hat{T}_2 = \frac{1}{\sqrt{37} \cdot \sqrt{17}} (-4)$

2. $6 \neq \cos(2\pi s) \implies$ curves do not intersect.

3. $t = s - 1$

$t^3 + 1 = s$

$0 = \sin(\pi s)$

$s = t + 1$

$t^3 + 1 = t + 1$

$t^3 - t = 0$

$\boxed{t = -1, 0, 1}$

$0 = \sin(-\pi)$ ✓

$0 = \sin(0)$ ✓

$0 = \sin(\pi)$ ✓

The curves intersect three times at $(-1, 0, 0)$ $(t=-1)$

$(0, 1, 0)$ $(t=0)$

$(1, 2, 0)$ $(t=1)$.

$\vec{r}_1{}'(t) = \langle 1, 3t^2, 0 \rangle$

$\hat{T}_1(t) = \frac{1}{\sqrt{1+9t^4}} \langle 1, 3t^2, 0 \rangle$

$\vec{r}_2{}'(t) = \langle 1, 1, \pi \cdot \cos(\pi s) \rangle$

$\hat{T}_2(t) = \frac{1}{\sqrt{2 + \pi^2 \cos^2(\pi s)}} \langle 1, 1, \pi \cos \pi s \rangle$

3. (cont).

$\hat{T}_1(-1) = \frac{1}{\sqrt{10}} \langle 1,3,0 \rangle$

$\hat{T}_2(0) = \frac{1}{\sqrt{2+\pi^2}} \langle 1,1,\pi \rangle$

$\cos(\theta_1) = \frac{1}{\sqrt{10(2+\pi^2)}} \cdot 4$

$\hat{T}_1(0) = \langle 1,0,0 \rangle$

$\hat{T}_2(1) = \frac{1}{\sqrt{2+\pi^2}} \langle 1,1,-\pi \rangle$

$\cos(\theta_2) = \frac{1}{\sqrt{(2+\pi^2)}} \cdot 1$

$\hat{T}_1(1) = \frac{1}{\sqrt{10}} \langle 1,3,0 \rangle$

$\hat{T}_2(2) = \frac{1}{\sqrt{2+\pi^2}} \langle 1,1,\pi \rangle$

$\cos(\theta_3) = \frac{1}{\sqrt{10(2+\pi^2)}} \cdot 4$

4. $\vec{r}'(t) = \langle -t, \sqrt{10t}, 5 \rangle$

$|\vec{r}'(t)| = \sqrt{t^2+10t+25} = t+5$

$\int_{-2}^{2} |\vec{r}'(t)|\,dt = \int_{-2}^{2} t+5 = \left[\frac{1}{2}t^2 + 5t\right]_{-2}^{2} = (2+10)-(2-10) = \boxed{20}$

5. $\vec{r}'(t) = \langle \frac{1}{t}, -\sqrt{2}, t \rangle$

$|\vec{r}'(t)| = \sqrt{\frac{1}{t^2}+2+t^2} = (t+\frac{1}{t})$

$\int_{1}^{e} t+\frac{1}{t} = \left[\frac{1}{2}t^2 + \ln(t)\right]_1^e = (\frac{1}{2}e^2 + 1) - (\frac{1}{2}+0) = \boxed{\frac{1}{2}e^2 + \frac{1}{2}}$

6. $\vec{r}'(t) = \langle -2e^{2t}, e^{2t}, -2e^{2t} \rangle$

$|\vec{r}'(t)| = \sqrt{4e^{4t} + e^{4t} + 4e^{4t}} = 3e^{2t}$

$\int_0^1 3e^{2t} = \left[\frac{3}{2}e^{2t}\right]_0^1 = \boxed{\frac{3}{2}e^2 - \frac{3}{2}}$

7. (a) $\vec{f}(t) = \langle -u, \frac{1}{2}u, -u \rangle$

(b) $1 \le u \le e^2$

(c) $\vec{f}\,'(t) = \langle -1, \frac{1}{2}, -1 \rangle$

$|\vec{f}\,'| = \sqrt{1 + \frac{1}{4} + 1} = \frac{3}{2}$

$\int_1^{e^2} \frac{3}{2}\, du = \left[\frac{3}{2}u\right]_1^{e^2} = \frac{3}{2}e^2 - \frac{3}{2}$

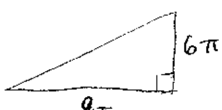
(d) yes. It is the same curve, so it has the same arc length.

8.(a) A circular spiral with radius

(b) a circle of radius 4

(c) Circumfrence $= 4\pi r = 8\pi$

(d) Starts at 20, ends at $20 - 6\pi$.
Change of 6π.

(e) 

if you coil up this right triangle, the bird is flying along the hypotenuse. so its a distace of 10π.

(f) $\vec{f}\,' = \langle -4\sin t, 4\cos t, -3 \rangle$

$|\vec{f}\,'| = \sqrt{16(\sin^2 t + \cos^2 t) + 9} = 5$

$\int_0^{2\pi} 5\, dt = [5t]_0^{2\pi} = 10\pi$