11-40

1. Write the line $\mathbf{r}(t) = 4\mathbf{i} - 2\mathbf{k} + t(-\mathbf{i} + 3\mathbf{j} + \mathbf{k})$ in parametric and symmetric form.

$x = 4 - t$

$y = 3t$

$z = -2 + t$

$$\frac{x-4}{-1} = \frac{y}{3} = z + 2$$

Determine whether each pair of lines intersects. If so, find the point of intersection and the angle formed by the lines.

2. $\mathbf{r}_1(t) = 2\mathbf{i} - \mathbf{j} + t(\mathbf{j} - \mathbf{k})$
   $\mathbf{r}_2(t) = 3\mathbf{i} + \mathbf{j} - \mathbf{k} + u(\mathbf{i} + \mathbf{k})$

$x = 2 = 3 + u$ $\quad u = -1$

$y = -1 + t = 1$ $\quad t = 2$

$z = -t = -1 + u$ $\quad -2 = -2$

Intersecting at point $(2, 1, -2)$

$\vec{d}_1 = \langle 0, 1, -1 \rangle$

$\vec{d}_2 = \langle 1, 0, 1 \rangle$

$$\cos\theta = \frac{\vec{d}_1 \cdot \vec{d}_2}{|\vec{d}_1||\vec{d}_2|} = \frac{-1}{\sqrt{2}\cdot\sqrt{2}} = -\frac{1}{2}$$

$$\theta = \frac{2}{3}\pi$$

3. $\mathbf{r}_1(t) = \mathbf{i} + \mathbf{j} + \mathbf{k} + t(2\mathbf{i})$
   $\mathbf{r}_2(t) = 4\mathbf{j} - \mathbf{k} + u(-\mathbf{i} + \mathbf{j})$

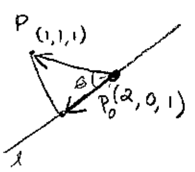
$x = 1 + 2t = -u$

$y = 1 = 4 + u$

$z = 1 = -1$

not intersecting since we can never have $1 = -1$.

4. Find the distance from the point $P(1, 1, 1)$ to the line, ℓ defined by $x = 2$, $y = t$, $z = 1 - t$



$$|\overrightarrow{P_0P}|\cdot\sin\theta = \frac{|\overrightarrow{P_0P}\times\vec{d}|}{|\vec{d}|} = \frac{\left\|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}\right\|}{|\langle 0, 1, -1\rangle|} = \frac{|-\hat{i} - \hat{j} - \hat{k}|}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}}$$

5. Find the equation of the plane orthogonal to $\langle 3, 1, -4\rangle$ and containing the point $P(3, 0, 9)$

$$3(x-3) + y - 4(z-9) = 0$$

6. A plane is given by the equation $3x + y - 2x = 4$. What vector is normal to this plane? Name any three points contained in this plane.

$\vec{N} = \langle 3, 1, -2 \rangle$

$(0, 0, -2)$

$(0, 4, 0)$

$(\frac{4}{3}, 0, 0)$

and others.

7. Consider the line ℓ given by $x = -2t + 1$, $y = t - 3$, $z = -t + 5$. The goal of this problem is to find a coinciding line, m where the equation for x is $x = u$.

(a) What is the direction vector of ℓ?

$\vec{d}_\ell = \langle -2, 1, -1 \rangle$

(b) What is the x component in the direction vector of m?

1

(c) What is the whole direction vector of m?

$\vec{d}_m = \langle 1, -\frac{1}{2}, \frac{1}{2} \rangle$

(d) Let P be the point used to specify m. What is the x coordinate of P?

0

(e) Since P is on m, it is also on ℓ. Use this fact to find all three coordinates of P.

$0 = -2t + 1$ $\quad y = \frac{1}{2} - 3 = -\frac{5}{2}$ $\quad z = -\frac{1}{2} + 5 = \frac{9}{2}$

$t = \frac{1}{2}$

(f) What are the parametric equations of m?

$x = u$ $\quad y = -\frac{5}{2} - \frac{1}{2}u$ $\quad z = \frac{9}{2} + \frac{1}{2}u$

(g) Verify your answer by finding two points that are contained in both ℓ and m.

in ℓ, $t = \frac{1}{2}$ $\quad$ in m, $u = 0$ $\quad$ in ℓ, $t = 0$ $\quad$ in m, $u = 1$

$(0, -\frac{5}{2}, \frac{9}{2})$ $\quad (0, -\frac{5}{2}, \frac{9}{2})$ $\quad (1, -3, 5)$ $\quad (1, -3, 5)$

Determine whether the following planes are identical, parallel, or intersecting. If intersecting, find the line where it intersects. (Hint: Since the line lies on both planes, it will be normal to both normal vectors. To find a point, guess one coordinate and solve for the other two.)

8. $x + y + z = 0$
$3x + 2y + z = 3$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{vmatrix} = \hat{i}(1-2) - \hat{j}(1-3) + \hat{k}(2-3) = -2\hat{i} + 2\hat{j} - \hat{k}.$$

if $x = 0$ $\quad y + z = 0$

$2y + z = 3$

$y = 3$, $z = -3$.

intersecting line

$x = -2t$

$y = 3 + 2t$

$z = -3 - t$

9. $2x + 6y - 6z = 2$
$\frac{x-1}{3} + y - z = 0$

$\vec{n}_1 = \langle 2, 6, -6 \rangle$

$\vec{n}_2 = \langle \frac{1}{3}, 1, -1 \rangle$

$\vec{n}_1 = 6\vec{n}_2$.

$6\left(\frac{x-1}{3} + y - z\right) = 6 \cdot 0$

$2x - 2 + 6y - 6z = 0$

$2x + 6y - 6z = 2$

$\boxed{\text{identical}}$

10. $2x - y + 8z = 10$
$-4x + 2y - 16z = 10$

$-\frac{1}{2}(-4x + 2y - 16z) = -\frac{1}{2} \cdot 10$

$2x - y + 8z = -5$

$\boxed{\text{parallel}}$